Unit 12: Circles, Arc Length, and Sector Area

Guided Notes

KEY

Name

Period

If found, please return to Mrs. Brandley's room, M-8.

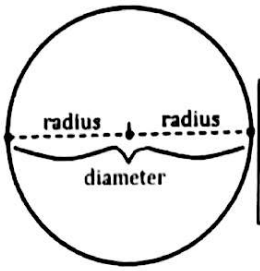
Concept 1: Circumference and Area

Circumference: The length of the outside of a circle

$C = 2\pi r$

Circle Area: Size of the surface of the circle

$A = \pi r^2$



Hint: The Diameter is 2x the radius!

Find the circumference of each circle with the given radius or diameter. Round to the nearest tenth, use 3.14 for π.

1. $r = 6\ cm$ $2\pi \times 6$

12π or $37.7\ cm$

2. $d = 47\ ft$ $r = 23.5$

$2\pi \times 23.5$

$147.7\ ft$

3. A pop can that has a radius of 1.5 in

$2\pi \times 1.5$

$9.4\ in$

Find the Area of each circle with the given radius or diameter. Round to the nearest tenth, Use 3.14 for π.

4. $r = 3\ mm$ $\pi \times 3^2$

9π or $28.3\ mm^2$

5. $d = 15\ yds$ $r = 7.5$

$\pi \times 7.5^2$

$176.7\ yd^2$

6. A CD has a diameter of 4.5 in $r = 2.25$

$\pi \times 2.25^2$

$15.9\ in$

Find the radius with the given circumferences and areas. Then find the circumference/area. Round to 2 decimal places.

7. $C = 43\ ft$ $2\pi r = 43$

$r = 6.8\ ft$

$A = 147.1\ ft^2$

8. $C = 97\ yd$ $2\pi r = 97$

$r = 15.4$

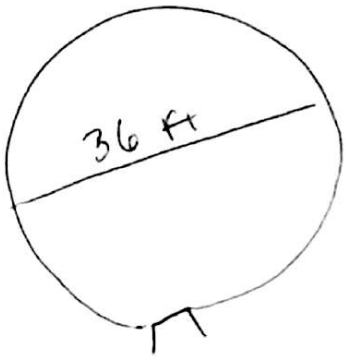
$A = 748.7\ yd^2$

9. $A = 92.75\ cm$ $\pi r^2 = 92.75$

$r = 5.4$

$A = 91.6\ cm^2$

10. A Ferris wheel has a diameter of 36 ft.



a) How far will an individual travel if the wheel rotates once?

$r = 18\ ft$ $2\pi(18)$

$113\ ft$

b) How far will they travel during a 2-minute ride if it rotates once every 20 seconds?

$6 \times 113 = 678\ ft$

Concept 2A: Arc Length

Arc Length: The distance from point A to point B, on the outside of a circle

radians = degrees × π / 180°

degrees = radians × 180° / π

Convert the following from degrees to radians, or radians to degrees.

1. 60°

1.05 or $\frac{\pi}{3}$

2. $\frac{\pi}{4}$ Radians

45°

3. 90°

$\frac{\pi}{2}$ or 1.6

4. 105°

1.8 or $\frac{7\pi}{12}$

5. π Radians

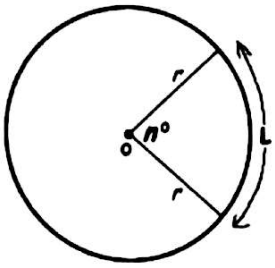
180°

6. $\frac{3\pi}{2}$ Radians

270°

Length of an Arc Formula

Length $= \frac{n°}{360°} \times 2\pi r$



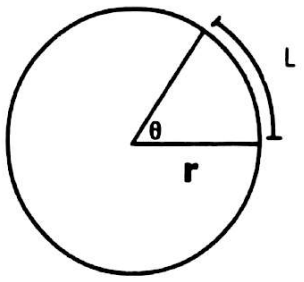
Example: r=7

n=80°

$\frac{80}{360} \times 2 \times \pi \times 7$

9.8

$L = r\theta$



Example: r=7

n=1.4 Radians

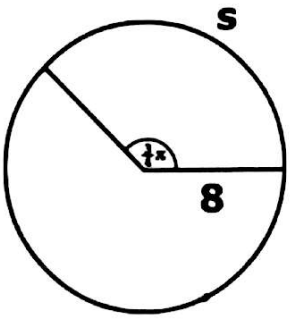
7 × 1.4

9.8

*****Notice that to find the length of an arc you just multiply the circumference of the circle by the fraction of the circle the length takes up.******

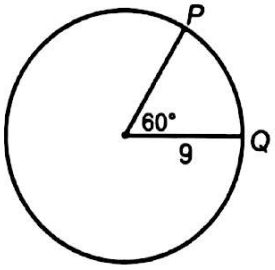
Find the arc length to the nearest tenth.

7.



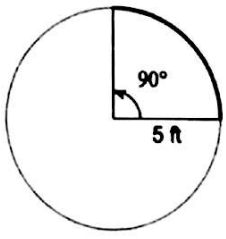
$\frac{3}{4}\pi \times 8 = 18.8$

8.



$\frac{60}{360} \times 2 \times \pi \times 9 = 3\pi$ or 9.4

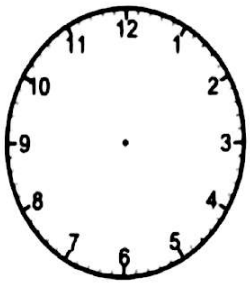
9.



A) 99.0 ft
B) 19.6 ft
C) 756.1 ft
D) 7.9 ft

10. The minute hand of a clock is 4 inches long. If the hand moves from 1:05 to 1:25, what is the distance the tip of the hand moves, to the nearest tenth? $r=4$ $120°$ $\frac{120}{360} \times 2 \times \pi \times 4 = 8.4$

Consider a standard 12-hour clock like the one below with a radius of 3 inches.
Use the shortest path between the two numbers.



11. What is the length of the arc between the 3 and the 8?

$r=3$ $150°$ 7.9 in

12. What is the length of the arc between the 3 and the 4?

$r=3$ $30°$ 1.6 in

13. It is 1:35. What is the length of the arc between the minute and hour hands?

$r=3$ $180°$ 3π or 9.4 in.

Concept 2B: Sector Area

Sector Area: The region within a circle bounded by two radii and an intercepted arc

Area of Sector

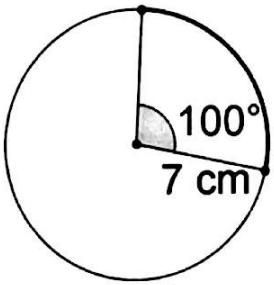
$A = \frac{\theta}{360°}\pi r^2$ where θ is in degrees

$A = \frac{1}{2}r^2\theta$ where θ is in radians

*****Notice that to find the area of a sector you just multiply the area of the circle by the fraction of the circle the sector takes up.******

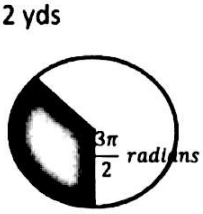
Find the shaded Sector area in the following circles.

1.



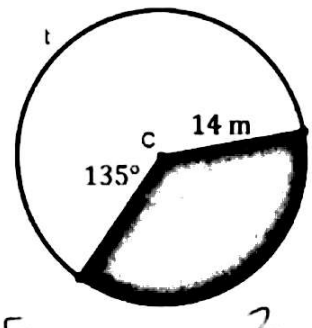
$\frac{100}{360} \times \pi \times 7^2 = 42.8 cm^2$

2.



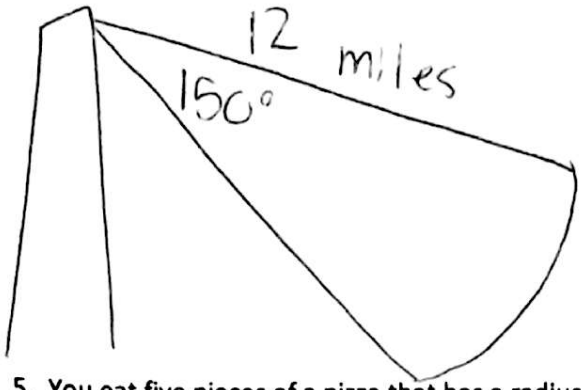
$\frac{1}{2} \times 2^2 \times \frac{3\pi}{2} = 3\pi$ or $9.4 yd^2$

3.



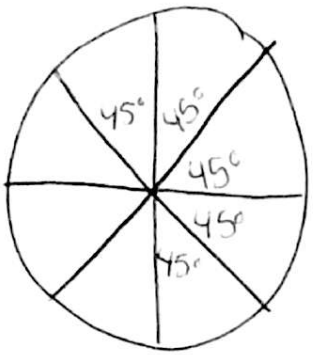
$\frac{135}{360} \times \pi \times 14^2 = 230.9 m^2$

4. The beam from a lighthouse is visible for a distance of 12 miles. What is the area covered when the beam sweeps in an arc of 150°? To the nearest tenth.



$\frac{150}{360} \times \pi \times 12^2 = 188.5 \text{ mi}^2$

5. You eat five pieces of a pizza that has a radius of 8 inches. The pizza is divided into eight even slices. What is the area of the pizza you ate?



$r = 8$

$360 \div 8 = 45°$

$225°$

$45 \times 5 = 225$

$\frac{225}{360} \times \pi \times 8^2 = 125.7 \text{ in}^2$

6. A large pizza has a radius of 9in. What is the area of half of the large pizza?

$\frac{180}{360} \times \pi \times 9^2 = 127.2 \text{ in}^2$ $r = 9$ $180°$

7. A slice is removed from a pizza with a radius of 7 inches. ~~The length of the crust of the missing slice is 2 in.~~ What is the area of the missing slice?

8 slices

$r = 7$ $45°$

$\frac{45}{360} \times \pi \times 7^2 = 19.2 \text{ in}^2$

Concept 3: Equations of Circles

Equation of a Circle: $(x-h)^2+(y-k)^2=r^2$

(h,k) is the center of the circle, r is the radius.

Identify the center and radius from equation.

1) $(x-9)^2+(y+11)^2=16$

C: (9, -11) r=4

2) $(x-1)^2+(y+10)^2=4$

C: (1, -10) r=2

3) $(x+3)^2+(y+15)^2=1$

C: (-3, -15) r=1

4) $(x-15)^2+(y+6)^2=6$

C: (15, -6) $r=\sqrt{6}$

Use center and radius to write equation.

5) Center: (–6, –1)
Radius: 4

$(x+6)^2+(y+1)^2=16$

6) Center: (15, 5)
Radius: 3

$(x-15)^2+(y-5)^2=9$

7) Center: (–1, 1)
Radius: 13

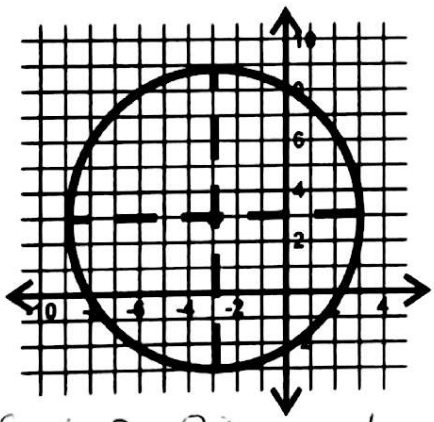
$(x+1)^2+(y-1)^2=169$

8) Center: (–15, –8)
Radius: 3

$(x+15)^2+(y+8)^2=9$

Write the equation of each circle.

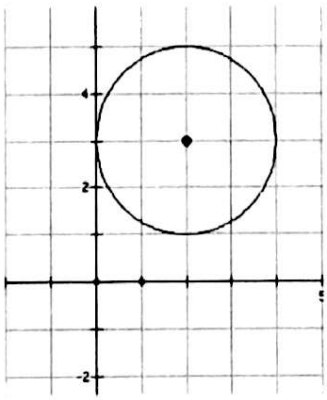
9.



C: (-3, 3) r=6

$(x+3)^2+(y-3)^2=36$

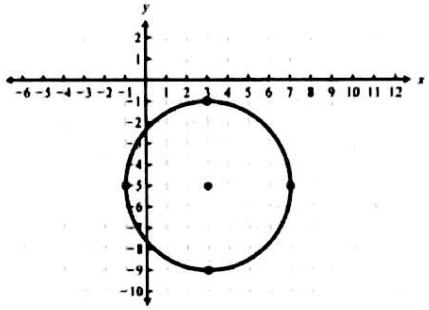
10.



C: (2, 3) r=2

$(x-2)^2+(y-3)^2=4$

11.



C: (3, -5) r=4

$(x-3)^2+(y+5)^2=16$

"Completing the Square"

1) Move constant to other side of the equal sign.
2) Put x terms together and y terms together in two sets of parentheses. Leave space.
3) Divide each b term by two and square it. Add the number(s) to both sides.
4) Write left side in factored form.

Identify the center and radius of the following.

1) $x^2 + y^2 + 16y + 39 = 0$

$x^2 + (y^2 + 16y + 64) = -39 + 64$

$x^2 + (y+8)^2 = 25$

C $(0, -8)$ $r = 5$

2) $x^2 + y^2 + 16x + 6y + 24 = 0$

$(x^2 + 16x + 64) + (y^2 + 6y + 9) = -24 + 64 + 9$

$(x+8)^2 + (y+3)^2 = 49$

C $(-8, -3)$ $r = 7$

3) $x^2 + y^2 - 18x - 16y + 64 = 0$

$(x^2 - 18x + 81) + (y^2 - 16y + 64) = -64 + 81 + 64$

$(x-9)^2 + (y-8)^2 = 81$

C $(9, 8)$ $r = 9$

4) $x^2 + y^2 + 14x - 8y + 16 = 0$

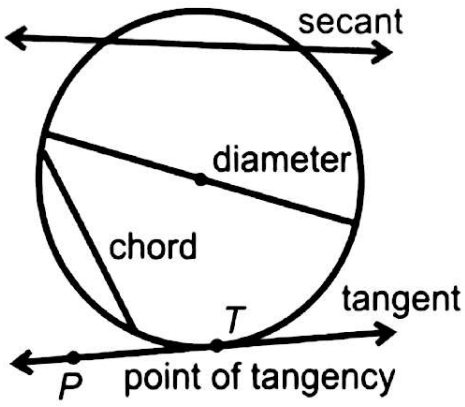
$(x^2 + 14x + 49) + (y^2 - 8y + 16) = -16 + 49 + 16$

$(x+7)^2 + (y-4)^2 = 49$

C $(-7, 4)$ $r = 7$

Concept 4: Circle Angles

Parts of a Circle



Diameter: Line from one side of the circle to the other that goes through the center

Radius: Line from the center to the outside of the circle. Half the diameter.

Secant: Line that goes through a circle intersecting at two points.

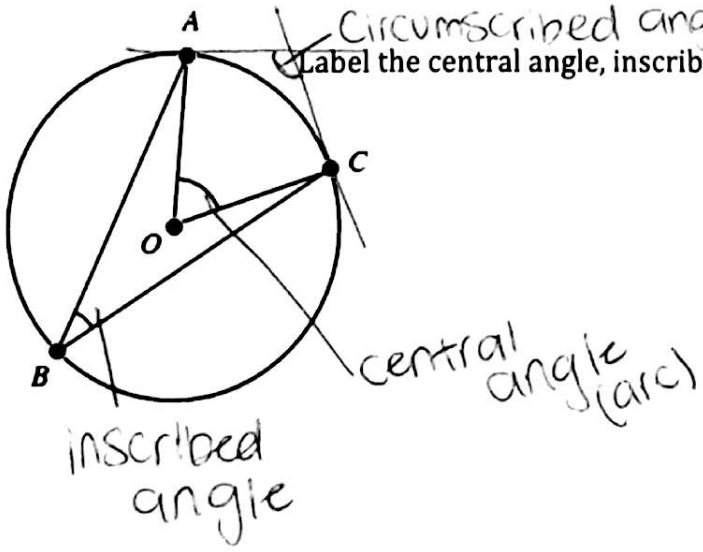
Chord: Line from a point on the edge of a circle to another point on the edge of the circle. The diameter is the longest chord.

Tangent: A line that intersects a circle only once on the edge of the circle.

Point of Tangency: Point where a tangent line intersects the circle.

Note that circles are congruent if they have the same radius. Think about when two circles would be similar, knowing that if one shape is a dilation of another, then the two are similar like we learned in our triangles unit.

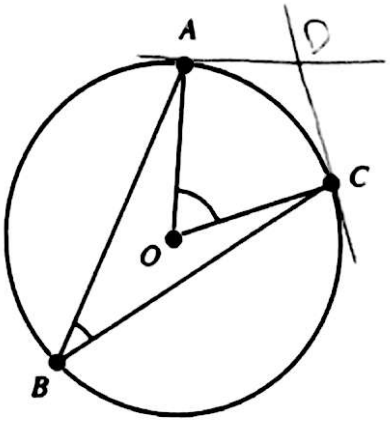
Label the central angle, inscribed angle, and circumscribed angle of the circle:



Central angle theorem: The central angle is always twice the measure of the inscribed angle.

Inscribed Angle Theorem: The inscribed angle is always half the angle.

Circumscribed Angle Theorem: The circumscribed angles is equal to 180 degrees minus the measure of the central angle.



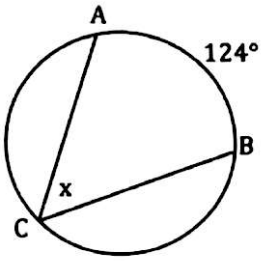
Given $\angle AOC = 60^\circ$

$\angle ADC = 120^\circ$

$\angle ABC = 30^\circ$

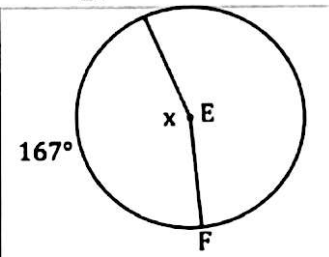
Using your knowledge of the Central angle and inscribed angle theorem, find the value for x.

1.



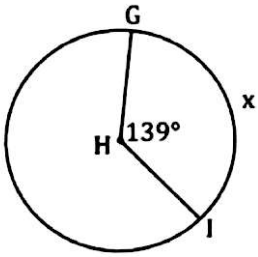
$X = 62^\circ$

2.



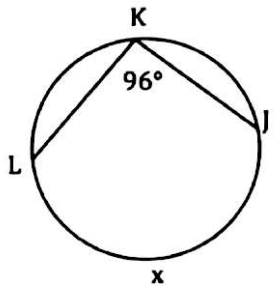
$X = 167^\circ$

3.



$X = 139^\circ$

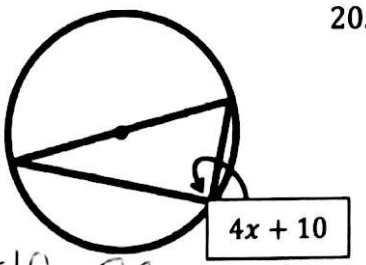
4.



$X = 192^\circ$

Using your knowledge of the Central angle and inscribed angle theorem, find the value for x.

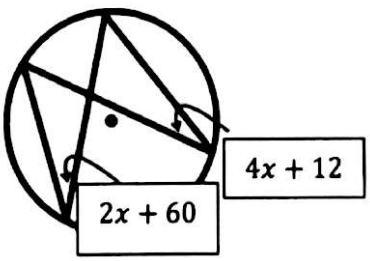
19.



$4x + 10 = 90$

$X = 20$

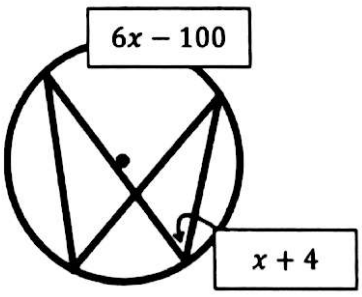
20.



$2x + 60 = 4x + 12$

$X = 24$

21.

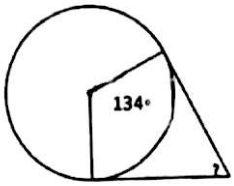


$2(x + 4) = 6x - 100$

$X = 27$

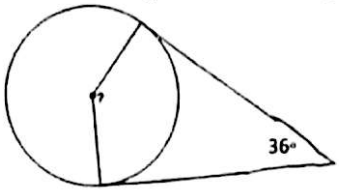
Solve for all the unknown angle and the angles formed by the tangent line and radius.

22.



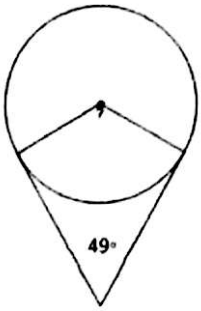
$180-134=46^\circ$

23.



$180-36=144^\circ$

24.



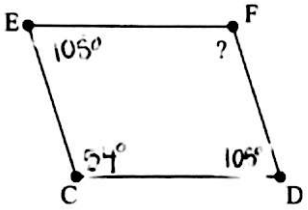
$180-49=131^\circ$

Concept 5: Inscribed Quadrilaterals

Quadrilaterals: Four sided figures whose angles add up to 360 degrees.

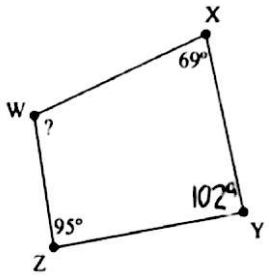
Find the measure of each indicated angle.

1.



$360-105-105-54=96^\circ$

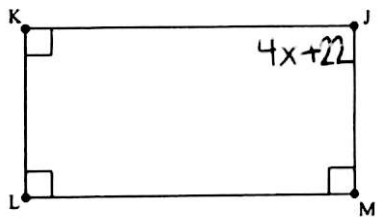
2.



$360-102-69-95=94^\circ$

Solve for x.

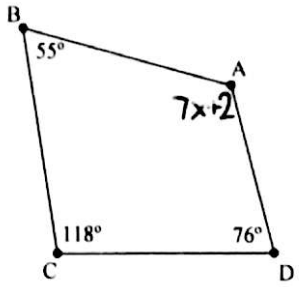
3.



$4x+22=90$

$x=17$

4.

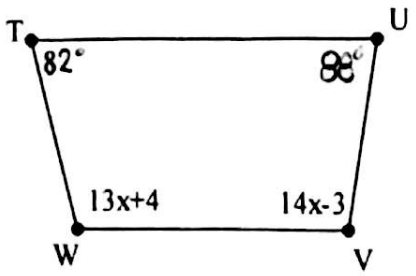


$360-55-118-76=111$

$7x+2=111$

$x=15.6$

5. Find the m∠V.

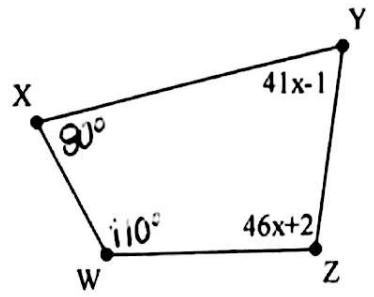


$360 - 82 - 88 = 190$

$13x + 4 + 14x - 3 = 190$

$x = 7 \quad m\angle V = 95°$

6. Find the m∠Z.



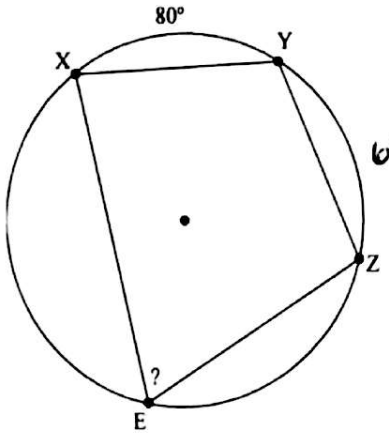
$360 - 80 - 110 = 170$

$41x - 1 + 46x + 2 = 170$

$x = 1.9 \quad m\angle Z = 91°$

Find the measure of **ALL** arcs or angles.

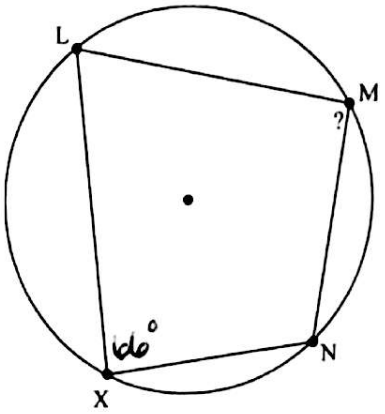
7. missing



$\frac{80+68}{2}$

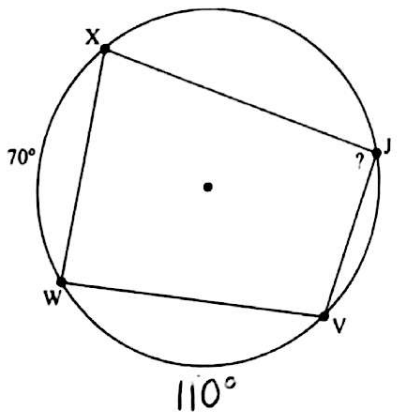
$74°$

8.



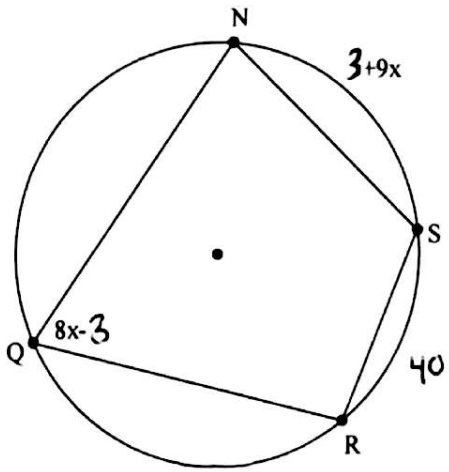
$180 - 66 = 114°$

9.



$\frac{110+70}{2} = 90°$

12. *Find* $m\widehat{NS}$.



$2(8x - 3) = 3 + 9x + 40$

$x = 7 \quad m\widehat{NS} = 66°$