

Unit 9: Probability

Guided Notes

KEY

Name

Period

If found, please return to Mrs. Brandley's room, M-8*

Concept 1: Set Notation

Sample Space: Set of all possible outcomes

Sample Size: Total number of possible outcomes

Sample Space for Choosing a Card from a Deck

Ex:

| | | | | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|----|------|-------|------|
| Ace | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Jack | Queen | King |
| ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ | ♥ |
| Ace | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Jack | Queen | King |
| ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ | ♦ |
| Ace | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Jack | Queen | King |
| ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ | ♠ |
| Ace | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Jack | Queen | King |
| ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ | ♣ |

Set: Collection of distinct elements

Subset: A set of which all the elements are contained in another set. Example: Set A: {1,2,3,4,5} and Set B: {1,3,5}. Thus $B \subset A$, which is said B is a subset of A.

Set Notation: {element, element, element, element}

Intersection: \cap OR AND

Union: \cup AND OR

Compliment: X^c NOT

Basic Probability:

1. P(3 of hearts)

$$\frac{1}{52}$$

$$= .02$$

$$2\%$$

2. P(jack)

$$\frac{4}{52} = \frac{1}{13}$$

$$.08$$

$$8\%$$

3. P(diamond)

$$\frac{13}{52} = \frac{1}{4}$$

$$.25$$

$$25\%$$

4. P(even number)

$$\frac{20}{52} = \frac{5}{13}$$

$$.38$$

$$38\%$$

5. P(face card)

$$\frac{12}{52} = \frac{3}{13}$$

$$.23$$

$$23\%$$

6. P(2 or 7)

$$\frac{8}{52} = \frac{2}{13}$$

$$.15$$

$$15\%$$

Use the following sets for the following questions. The SAMPLE SPACE is the numbers 1 thru 20

Write out the sample space here: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

E Set A is the set of odd numbers.

Write out Set A here: $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

F Set B is the set of even numbers.

Write out Set B here: $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

G Set C is the multiples of 5 less than 21.

Write out Set C here: $\{5, 10, 15, 20\}$

Set D is all prime numbers less than 21.

Write out Set D here: $\{2, 3, 5, 7, 11, 13, 17, 19\}$

1. Is $B \subset A$?

no

2. Is $D \subset B$?

no

3. Is $D \subset A$?

~~yes~~ no

4. What is $D \cup E$?

$\{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

5. What is $D \cap E$?

$\{3, 5, 7, 11, 13, 17, 19\}$

6. What is $E \cap G$?

$\{5, 15\}$

7. What is $F \cup G$?

$\{2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20\}$

8. What is D^c ?

$\{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

9. What is E^c ?

$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

10. What is $P(D)$?

$$8/20 = 2/5$$

$$.4 \quad 40\%$$

11. What is $P(G)$?

$$4/20 = 1/5$$

$$.2 \quad 20\%$$

12. What is $P(G \cup F)$?

$$12/20 \quad 3/5 \quad .6 \quad 60\%$$

13. What is $P(E \cap G)$?

$$2/20 \quad 1/10 \quad .1 \quad 10\%$$

A Sweet Task – A Lesson in Exploring Conditional Probability Activity Sheet

Name _____ Date _____

One member of each group should obtain a bag of M&Ms and a bag of Skittles from the teacher.

1. Collect Data: Count the number of M&Ms and Skittles of each color in your bag and record this information in the frequency tables below. Then, either enter the data on your teacher's computer or give the data to your teacher.

| | Red | Orange | Yellow | Green | Blue | Brown | Total |
|-------|-----|--------|--------|-------|------|-------|-------|
| M&M's | | | | | | | |

| | Red | Orange | Yellow | Green | Blue | Purple | Total |
|-----------|-----|--------|--------|-------|------|--------|-------|
| Skittle's | | | | | | | |

2. Create a Two-Way Frequency Table: We can combine individual frequency tables into a two-way frequency table. The rows represent the **types of candy** and the columns represent the **color of the candy**. Use the combined class data (provided by your teacher) to fill in the two-way frequency table below. **Be sure to total each column and row.**

| | Red | Orange | Yellow | Green | Blue | Purple | Brown | Total |
|-----------|-----|--------|--------|-------|------|--------|-------|-------|
| M&M's | 46 | 58 | 45 | 49 | 80 | 0 | 48 | 326 |
| Skittle's | 50 | 48 | 47 | 66 | 0 | 43 | 0 | 254 |
| Total | 96 | 106 | 92 | 115 | 80 | 43 | 48 | 580 |

We read a two-way frequency table in a similar way as a regular frequency table. For example, the number of orange Skittles is listed where the "Orange" column and the "Skittles" row meet. This is called a **joint frequency**.

We can also find the total number of blue candies in the bag. We just look at the total of the "Blue" column. This is a **marginal frequency**.

Checkpoint #1

3. Analyzing the Data – Finding Marginal and Joint Probabilities: We can compute the probability of an event occurring from the frequency counts within the candy “mix” two-way frequency table. Find the probability of randomly choosing a candy with the listed attributes. Also, identify each event as either a joint or marginal probability. A joint probability requires two or more characteristics to hold true, whereas a marginal probability requires only one.

| | <u>Probability</u> | <u>Joint or Marginal Probability</u> |
|----------------------|-----------------------|--------------------------------------|
| a. Any Color M&M | $\frac{326}{580}$ 56% | marginal |
| b. A Purple Skittle | $\frac{43}{580}$ 7% | joint |
| c. A Blue M&M | $\frac{80}{580}$ 14% | joint |
| d. An Orange Skittle | $\frac{48}{580}$ 8% | joint |
| e. Any Green candy | $\frac{115}{580}$ 20% | marginal |
| f. A Blue Skittle | $\frac{0}{580}$ 0% | joint |

g. In your own words describe how you compute a joint probability given counts in a two-way frequency table.

Find the # of candies w/ Both characteristics and divide it by the total # of candies.

h. In your own words describe how you compute a marginal probability given counts in a two-way frequency table.

Find the # of candies w/ that one characteristic and divide it by the

Checkpoint #2

total # of characteristics.

4. Finding Conditional Probability with Counts: Imagine that your friend chooses a candy piece from the class "mix". She looks at it, tells you that it is red, but doesn't tell you if it is an M&M or a Skittle.

Knowing that your friend has a red candy in her hand, we can find the probability that it is a red M&M. This is called the **conditional probability** of an event because **we already know something (a condition) about the event in question.**

Answer the following questions to help you find the conditional probability.

a. What is the "total number of possible outcomes" for your friend's candy? (*Remember we know the candy is red.*)

96

b. What is the probability of your friend having an M&M, if we know the candy is red? (*Keep in mind we only are worried about M&Ms that are red.*)

46/96 48%

c. In your own words, explain how to compute conditional probabilities given a two-way frequency table.

Your total is based on what is given (ex total of reds) and then your top # is based on how many of what you're looking for that's red (ex red m&ms)

What you just found can be written as $P(\text{M\&M} \mid \text{red})$, which we read as "the probability of a candy being an M&M *given* that it is red".

5. Computing Conditional Probabilities: Using the class data about the candy "mix", find the following conditional probabilities.

a. $P(\text{green} \mid \text{Skittle})$

$$\frac{66}{254} \quad 26\%$$

b. $P(\text{M\&M} \mid \text{blue})$

$$\frac{80}{80} \quad 100\%$$

c. $P(\text{brown} \mid \text{M\&M})$

$$\frac{48}{326} \quad 15\%$$

d. $P(\text{Skittle} \mid \text{red})$

$$\frac{50}{96} \quad 52\%$$

e. $P(\text{Skittle} \mid \text{purple})$

$$\frac{43}{43} \quad 100\%$$

f. $P(\text{M\&M} \mid \text{purple})$

$$\frac{0}{43} \quad 0\%$$

g. $P(\text{yellow} \mid \text{M\&M})$

$$\frac{45}{326} \quad 14\%$$

h. $P(\text{M\&M} \mid \text{yellow})$

$$\frac{45}{92} \quad 49\%$$

Checkpoint #3

If you draw a red candy, is it more likely to be an M&M or Skittle? Why?

A skittle because there are more red skittles than there are red m&ms.

Formula for Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

| | Plan to Vacation in New York Within the Next Year | Do Not Plan to Vacation in New York Within the Next Year | Total |
|-----------------------------|---|--|-------|
| Watched the Online Ad | 300 | 450 | 750 |
| Did Not Watch the Online Ad | 100 | 150 | 250 |
| Total | 400 | 600 | 1,000 |

1. What is the probability someone plans a vacation to NY within a year if they watch the online ad?

$$\frac{300}{750} \quad 40\%$$

2. What is the probability that someone plans a vacation to NY within a year if they do not watch the online ad?

$$\frac{100}{250} \quad 40\%$$

3. Do you think the ad works? Why or why not?

No the probability of them planning a trip to NY is the same whether they watch the ad or not

4. A box contains three blue marbles, five red marbles, and four white marbles. If one marble is drawn at random, find:

a) $P(\text{blue}|\text{not White}) \quad 3/8 \quad 38\%$

b) $P(\text{not Red}|\text{not White}) \quad 3/8 \quad 38\%$

5. A number is selected randomly from a container containing all the integers from 10 to 50. Find:

a) $P(\text{even}|\text{greater than 40}) \quad 5/10 \quad 50\%$

b) $P(\text{greater than 40}|\text{even}) \quad 5/21 \quad 24\%$

c) $P(\text{prime}|\text{between 20 and 40}) \quad 4/21 \quad 19\%$

Concept 3: Reasoning with Probability

Answer the following questions and use probability to defend your answer.

1. Dylan and Cade split the cost of a package of five passes to a climbing gym. Describe a way that you could fairly decide who gets to use the fifth pass.

- Flip a coin assigning one heads & tails
- Roll a dice assign one evens & one odds

2. In addition to prizes for first, second, and third place, the organizers of a race have a prize that they want each participant to have an equal chance of winning. Describe a fair method of choosing a winner for this prize.

- Putting everyone's name in a raffle and drawing one out.

3. Two teams are playing a game against one another in class to earn 10 extra points on an assignment. The teacher said that the points will be split fairly between the two teams, depending on the results of the game. If Team A earned 1300 points and Team B earned 2200 points describe one way the teacher could split up the 10 extra points.

$$\text{Team A } \frac{1300}{3500} = 37\% \quad \text{Team B } \frac{2200}{3500} = 63\%$$

$$\text{Team A: } 3.7 \text{ points} \quad \text{Team B: } 7$$

4. Joslin and McCall are at a yard sale, and they find a box of 20 collectible toys they both want. They can't agree about who saw it first, so they flip a coin until Alexa gets 10 heads or Sofia gets 10 tails. When Alexa has 3 heads and Sofia has 6 tails, they decide to divide the toys proportionally based on the probability each has of winning under the original rules. How should they divide the toys?

$$A: 3/10 : 30\% \quad S: 6/10 : 60\%$$

Alexa gets 6 of the toys (30% of them)
Sofia gets 12 of the toys (60% of them). Then they each get 1 of the last 2!

5. Bernard and Braxton are fighting over who gets to pick where to go to lunch. They decide to roll a die to decide who gets to choose. Bernard says he gets to choose if the they roll less than a 4 and Braxton gets to roll if it's greater than 4. Is this a fair way to choose? Why or why not?

No. Bernard has a 50% chance of getting to choose (3/6) and Braxton only has a 33% chance (2/6)