Unit 7: Solving Quadratics

Guided Notes

KEY
Name

Period

If found, please return to Mrs. Brandley's room, M-8.

Scanned by CamScanner

Concept 1 WARM-UP

Solve the following equations for x. Simplify the radicals

1. $x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

2. $x^2 = 36$

$x = \pm\sqrt{36}$

$x = \pm 6$

3. $x^2 = -16$

$x = \pm\sqrt{-16}$

$x = \pm 4i$

4. $x^2 = -49$

$x = \pm\sqrt{-49}$

$x = \pm 7i$

Simplify the following radicals.

5. $\sqrt{20}$

$\sqrt{5 \cdot 2 \cdot 2}$

$\boxed{2\sqrt{5}}$

6. $\sqrt{60}$

$\sqrt{5 \cdot 2 \cdot 3 \cdot 2}$

$\boxed{2\sqrt{15}}$

7. $\sqrt{-80}$

$= i\sqrt{80}$

$\boxed{4i\sqrt{5}}$

Concept 1: The Square Root Method

Remember that all quadratics have two solutions. Either one repeated, 2 real, or 2 complex.

Solve the following equations for x.

No x-term:

8. $x^2 - 81 = 0$

$x^2 = 81$

$\boxed{x = \pm 9}$

9. $x^2 + 64 = 0$

$x^2 = -64$

$\boxed{x = \pm 8i}$

10. $x^2 - 60 = 0$

$x^2 = 60$

$\boxed{x = \pm 2\sqrt{15}}$

11. $2x^2 - 64 = 0$

$2x^2 = 64$

$x^2 = 32$

$\boxed{x = \pm 4\sqrt{2}}$

12. $3x^2 + 81 = 0$

$3x^2 = -81$

$x^2 = -27$

$\boxed{x = \pm 3i\sqrt{3}}$

13. $4x^2 - 64 = 0$

$4x^2 = 64$

$x^2 = 16$

$\boxed{x = \pm 4}$

Vertex Form:

14. $(x-1)^2 - 9 = 0$

$(x-1)^2 = 9$

$x - 1 = \pm 3$

$\boxed{x = 4, -2}$

15. $3(x-2)^2 + 27 = 0$

$3(x-2)^2 = -27$

$(x-2)^2 = -9$

$x - 2 = \pm 3i$

$\boxed{x = 2 \pm 3i}$

16. $4(x+3)^2 - 16 = 0$

$4(x+3)^2 = 16$

$(x+3)^2 = 4$

$x + 3 = \pm 2$

$\boxed{x = 5, 1}$

Scanned by CamScanner

Concept 2: WARM-UP

Use the distributive property to simplify the following quadratics in standard form.

1. $(x+2)(x-5)$

$x^2+2x-5x-10$

$x^2-3x-10$

2. $(x+4)^2$ $(x+4)(x+4)$

$x^2+8x+16$

3. $3(x+1)(x-6)$

$3(x^2+x-6x-6)$

$3(x^2-5x-6)$

$3x^2-15x-18$

Concept 2: Factoring "Undistributing"

Factoring in essence is "undistributing". Going from standard form to factored form. Every "factorable" problem can be factored using the following steps.

1. Write the quadratic in standard form. Ax^2+Bx+C
2. Factor out the greatest common factor (GCF). (There will not always be one).
3. Find two numbers that multiply to AC and add to B.
4. Rewrite the quadratic splitting up the x-term into two terms using the two numbers found in step 3.
5. Group two sides of the quadratic in a way that makes the parentheses match.
6. Factor out the term in parentheses and rewrite in factored form.
7. Smile in relief that your problem is complete. ☺

Example: $-2+4x^2=6x$

1. $4x^2-6x-2=0$
2. $2(2x^2-3x-2)$
3. AC=-4 B=-3. -4 and 1 multiply to -4 and add to -3.
4. $2(2x^2-4x+1x-2)$
5. $2(2x(x-2)+1(x-2))$
6. $2(2x+1)(x-2)$

Scanned by CamScanner

Let's try it a step at a time!

Example 1: $-10 + x^2 = -3x$

1. Write the quadratic in standard form. $Ax^2 + Bx + C$

$x^2 + 3x - 10 = 0$

2. Factor out the greatest common factor (GCF). (There will not always be one).

none

3. Find two numbers that multiply to AC and add to B.

$AC = -10 \quad B = 3 \qquad -2 \,\&\, 5$

4. Rewrite the quadratic splitting up the x-term into two terms using the two numbers found in step 3.

$x^2 - 2x + 5x - 10$

5. Group two sides of the quadratic in a way that makes the parentheses match.

$x(x-2) + 5(x-2)$

6. Factor out the term in parentheses and rewrite in factored form.

$(x-2)(x+5)$

7. Smile in relief that your problem is complete. ☺

Example 2: $8x + x^2 = -16$

1. Write the quadratic in standard form. $Ax^2 + Bx + C$

$x^2 + 8x + 16 = 0$

2. Factor out the greatest common factor (GCF). (There will not always be one).

none

3. Find two numbers that multiply to AC and add to B.

$AC = 16 \quad B = 8 \qquad 4 \,\&\, 4$

4. Rewrite the quadratic splitting up the x-term into two terms using the two numbers found in step 3.

$x^2 + 4x + 4x + 16$

5. Group two sides of the quadratic in a way that makes the parentheses match.

$x(x+4) + 4(x+4)$

6. Factor out the term in parentheses and rewrite in factored form.

$(x+4)(x+4) = (x+4)^2$

7. Smile in relief that your problem is complete. ☺

Scanned by CamScanner

Example 3: $3x^2 - 15x - 18 = 0$

1. Write the quadratic in standard form. $Ax^2 + Bx + C$

$3x^2 - 15x - 18 = 0$

2. Factor out the greatest common factor (GCF). (There will not always be one).

$3(x^2 - 5x - 6) = 0$

3. Find two numbers that multiply to AC and add to B.

$AC = -6 \quad B = -5 \qquad -6 \,\&\, 1$

4. Rewrite the quadratic splitting up the x-term into two terms using the two numbers found in step 3.

$3(x^2 + x - 6x - 6)$

5. Group two sides of the quadratic in a way that makes the parentheses match.

$3(x(x+1) - 6(x+1))$

6. Factor out the term in parentheses and rewrite in factored form.

$3(x+1)(x-6)$

7. Smile in relief that your problem is complete. ☺

As a class: $2x^2 - 4x = -2$

$2x^2 - 4x + 2 = 0$

$2(x^2 - 2x + 1) = 0$

$2(x^2 - x - x + 1)$

$2(x(x-1) - 1(x-1))$

$2(x-1)(x-1) = 2(x-1)^2$

With a partner: $\cancel{-}9 - 6x = x^2$

$x^2 + 6x + 9 = 0$

$x^2 + 3x + 3x + 9$

$x(x+3) + 3(x+3)$

$(x+3)(x+3)$

Shoulder Tap: $x^2 - x - 2 = 0$

$x^2 + x - 2x - 2$

$x(x+1) - 2(x+1)$

$(x-2)(x+1)$

Concept 2: WARM-UP 2

Find the solutions of the following:

1. $(x+3)(3x-9) = 0$

$x + 3 = 0 \qquad 3x - 9 = 0$

$\boxed{x = -3} \qquad 3x = 9$

$\boxed{x = 3}$

2. $2(2x+4)(x-2) = 0$

$2x + 4 = 0 \qquad x - 2 = 0$

$2x = -4 \qquad \boxed{x = 2}$

$\boxed{x = -2}$

3. $3x(x+1)(2x-8) = 0$

$3x = 0 \qquad x + 1 = 0 \qquad 2x - 8 = 0$

$\boxed{x = 0} \qquad \boxed{x = -1} \qquad 2x = 8$

$\boxed{x = 4}$

Scanned by CamScanner

Concept 2 Day 2: Solving by Factoring

Factor the following quadratics and then find the solutions.

As a class:

1. $2x^2 - 8 = 6x$ $\boxed{x=-1, 4}$

$2x^2-6x-8$

$2(x^2-3x-4)$

$2(x^2+x-4x-4)$

$2(x(x+1)-4(x+1))$

$2(x+1)(x-4)$

2. $2x^2 + 7x = -3$ $\boxed{x=-\frac{1}{2}, -3}$

$2x^2+7x+3=0$

$2x^2+x+6x+3=0$

$x(2x+1)+3(2x+1)$

$(2x+1)(x+3)$

3. $6x^2 + 5x = +6$ $\boxed{x=\frac{2}{3}, -\frac{3}{2}}$

$6x^2+5x+6$

$6x^2-4x+9x-6$

$2x(3x-2)+3(3x-2)$

$(3x-2)(2x+3)$

With a partner:

4. $2x^2 + 36 = 18x$ $\boxed{x=6, 3}$

$2x^2-18x+36$

$2(x^2-9x+18)$

$2(x^2-3x-6x+18)$

$2(x(x-3)-6(x-3))$

$\boxed{2(x-6)(x-3)}$

5. $x^2 + 2x = -1$ $\boxed{x=-1}$

x^2+2x+1

$x^2+x+x+1$

$x(x+1)+1(x+1)$

$(x+1)(x+1)$

$\boxed{(x+1)^2}$

6. $2x^2 + 11x - 21 = 0$ $\boxed{x=\frac{3}{2}, -7}$

$2x^2-3x+14x-21$

$x(2x-3)+7(2x-3)$

$(2x-3)(x+7)$

Shoulder Tap:

7. $x^2 + 4x + 4 = 0$ $\boxed{x=-2}$

$x^2+2x+2x+4$

$x(x+2)+2(x+2)$

$(x+2)(x+2)$

$\boxed{(x+2)^2}$

8. $2x^2 + 18 = -6x$

$2x^2+6x+18$

$2(x^2+3x+9)$

9. $2x^2 - 3 = -5x$ $\boxed{x=\frac{1}{2}, -3}$

$2x^2+5x-3$

$2x^2-x+6x-3$

$x(2x-1)+3(2x-1)$

$(2x-1)(x+3)$

Concept 3: WARM-UP

Put the following quadratics in standard form and list a, b, and c.

1. $x^2 = 5x - 3$

SF: x^2-5x+3

A: 1

B: -5

C: 3

2. . $2x^2 = 5 - 4x$

SF: $2x^2+4x-5$

A: 2

B: 4

C: -5

3. $2x = -6x^2 - 1$

SF: $-6x^2-2x-1$

A: -6

B: -2

C: -1

Scanned by CamScanner

Concept 3: The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula above is the quadratic formula. It is much less scary than it looks. It is just a combination of the axis of symmetry and discriminant like you have used before! The cool thing about the quadratic formula is that it can be used to solve ALL quadratics including those with complex solutions AND an x term. Let's try it!

As a class:

1. $4x^2 + 12x = 7$

$4x^2+12x-7$

$$x=\frac{-12\pm\sqrt{12^2-4(4)(-7)}}{2(4)}$$

$$\boxed{x=-\frac{7}{2},\ \frac{1}{2}}$$

2. $4x^2 + 3 = -6x$

$4x^2+6x+3$

$$x=\frac{-6\pm\sqrt{6^2-4(4)(3)}}{2(4)}$$

$$\boxed{x=\frac{-3\pm i\sqrt{3}}{4}}$$

3. $3x^2 + 3 = -2x$

$3x^2+2x+3$

$$x=\frac{-2\pm\sqrt{2^2-4(3)(3)}}{2(3)}$$

$$\boxed{x=\frac{-1\pm 2i\sqrt{2}}{3}}$$

With a partner:

4. $2x^2 - 5x = 75$

$2x^2-5x-75$

$$x=\frac{5\pm\sqrt{(-5)^2-4(2)(-75)}}{2(2)}$$

$$\boxed{x=\frac{29}{4},\ -5}$$

5. $3x^2 + 6 = -72$

$3x^2+0x+72$

$$x=\frac{\pm\sqrt{0^2-4(3)(72)}}{2(3)}$$

$$\boxed{x=\pm 2i\sqrt{6}}$$

6. $2x^2 + 1 = 2x$

$2x^2-2x+1$

$$x=\frac{2\pm\sqrt{(-2)^2-4(2)(1)}}{2(2)}$$

$$\boxed{x=\frac{1\pm i}{2}}$$

Shoulder Tap:

7. $4x^2 + 9x = 13$

$4x^2+9x-13$

$$x=\frac{-9\pm\sqrt{9^2-4(4)(-13)}}{2(4)}$$

$$\boxed{x=1,\ -\frac{13}{4}}$$

8. $x^2 = -8x + 19$

$x^2+8x-19$

$$x=\frac{-8\pm\sqrt{(8)^2-4(1)(-19)}}{2(1)}$$

$$\boxed{x=4\pm\sqrt{35}}$$

9. $x^2 = 4 + x$

x^2-x-4

$$x=\frac{1\pm\sqrt{(-1)^2-4(1)(-4)}}{2(1)}$$

$$\boxed{x=\frac{-1\pm i\sqrt{15}}{2}}$$

Scanned by CamScanner