Unit 7 Graphing Rational Equations

Guided Notes

KEY

Name

Period

If found, please return to Mrs. Brandley's room, M-8.

Concept 1: Identify domain, range, holes, and asymptotes from a graph.

Vertical Asymptote: Vertical line that the graph approaches but never touches.

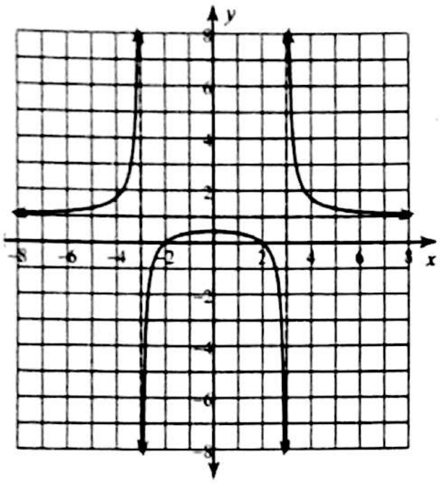
Horizontal Asymptote: Horizontal line that the graph approaches but never touches.

Hole: Point on the graph where the function isn't defined. (shown with an open circle)

Domain: All possible x-values for a given function.

Range: All possible y-values for a given function.

$$f(x) = \frac{x^2 - 4}{x^2 - 9}$$



V.A.(s): $x = -3, 3$

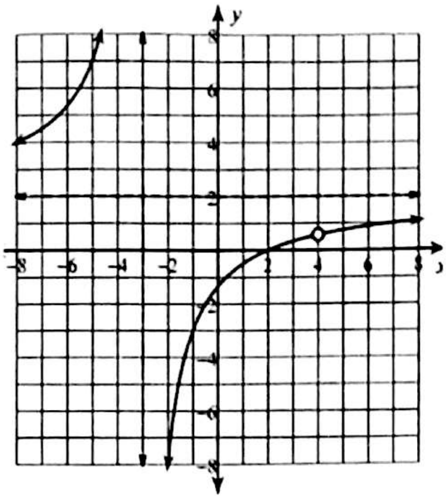
H.A.(s): $y = 1$

Hole(s): none

Domain: $\mathbb{R}$ $x \neq -3, 3$

Range: $\mathbb{R}$ $y \neq 1$

$$f(x) = \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$



V.A.(s): $x = -3$

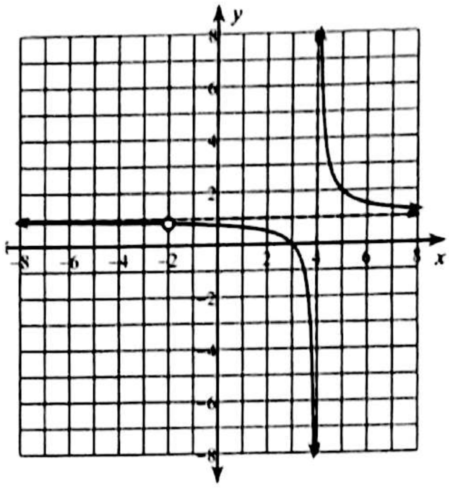
H.A.(s): $y = 2$

Hole(s): $x = 4$

Domain: $(-\infty, \infty)$ $x \neq -3, 4$

Range: $(-\infty, \infty)$ $y \neq 1/2, 2$

$$f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 8}$$



V.A.(s): $x = 4$

H.A.(s): $y = 1$

Hole(s): $x = -2$

Domain: $\mathbb{R}$ $x \neq -2, 4$

Range: $\mathbb{R}$ $y \neq 8, 1$

Concept 2: Identify Domain, Holes, and Asymptotes from an equation.

Horizontal Asymptote: Horizontal line the graph of a function approaches but never touches.

To find the horizontal asymptote:

- If the highest exponent in the numerator is higher than the highest one in the denominator:
  - NO horizontal asymptote. EX: $\frac{x^3-5x^2+7}{x^2-4x+5}$ HA: none
- If the highest exponent in the denominator is higher than the highest one in the numerator
  - Horizontal asymptote at y=0. EX: $\frac{x^2-5x+7}{x^3-4x+5}$ HA: y=0
- If the highest exponents are the same in the numerator and denominator:
  - Horizontal asymptote at y=the ratio of leading coefficients from the numerator and denominator. EX: $\frac{4x^2-5x+7}{2x^2-4x+5}$ HA: $y=\frac{4}{2}=2$

Find the horizontal asymptote of the following:

1) $f(x)=\dfrac{x^3+x^2-2x}{-3x^3+12x^2-9x}$

$y=-\frac{1}{3}$

2) $f(x)=\dfrac{x^3+x^2-2x}{-4x^2+8x+12}$

none

3) $f(x)=\dfrac{x^2+2x}{x^3-x^2-6x}$

$y=0$

4) $f(x)=\dfrac{x+4}{x-1}$

$y=1$

Vertical Asymptote: Vertical line a graph approaches but never touches. To find the vertical asymptote(s) from a function's equation, you simply set the denominator equal to 0. (Similar to finding excluded values.) Find the vertical asymptotes for the following two examples.

1) $f(x)=\dfrac{x^3-x^2-2x}{4x^2-12x}$ $=4x(x-3)$

$4x=0$ $x-3=0$

$x=0$ $x=3$

$x=0,3$

2) $f(x)=-\dfrac{3x}{x^2+x-2}$ $=(x-1)(x+2)$

$x-1=0$ $x+2=0$

$x=1$ $x=-2$

$x=1,-2$

Hole: Point on the graph where the function is not defined.

Any factors that cancel out from the top and bottom create a hole instead of a vertical asymptote.

Find the vertical asymptotes and holes of the following:

1) $f(x) = \dfrac{x^3 - 3x^2 + 2x}{4x^2 + 4x} = \dfrac{x(x^2-3x+2)}{4x(x+1)}$

$= \dfrac{\cancel{x}(x-2)(x-1)}{4\cancel{x}}$

Hole $x=0$

V.A none

Domain: $\mathbb{R}$, $x \neq 0$

2) $f(x) = \dfrac{4x-4}{x^2+3x-4} = \dfrac{4(x-\cancel{1})}{(x-\cancel{1})(x+3)}$

Hole: $x=1$

V.A: $x=-3$

Domain $\mathbb{R}$ $x \neq 1, -3$

3) $f(x) = \dfrac{x^2+2x}{x^3-x^2-6x} = \dfrac{x(x+2)}{x(x^2-x-6)}$

$= \dfrac{\cancel{x}(x+\cancel{2})}{\cancel{x}(x-3)(\cancel{x+2})}$

Holes $x=0, -2$

V.A. $x=3$

Domain $\mathbb{R}$ $x \neq 0, -2, 3$

4) $f(x) = \dfrac{x^3-x^2-6x}{-3x^2+3x} = \dfrac{x(x^2-x-6)}{-3x(x-3)}$

$= \dfrac{x(\cancel{x-3})(x+2)}{-3\cancel{x}(\cancel{x-3})}$

Holes: $x=0, 3$

V.A.: none

Doman $\mathbb{R}$ $x \neq 0, 3$

Fun Fact! Vertical Asymptote trumps hole! Watch: $\dfrac{(x-5)(\cancel{x+2})}{(\cancel{x+2})(x+2)}$ — V.A. $x=-2$

Domain: All possible x-values of a given function.

In the case of rational equations, the domain includes all real numbers EXCEPT anywhere there is a vertical asymptote or a hole.

State the domain ~~of the following~~ of the four rational functions above.

Concept 3: Graphing Rational Equations

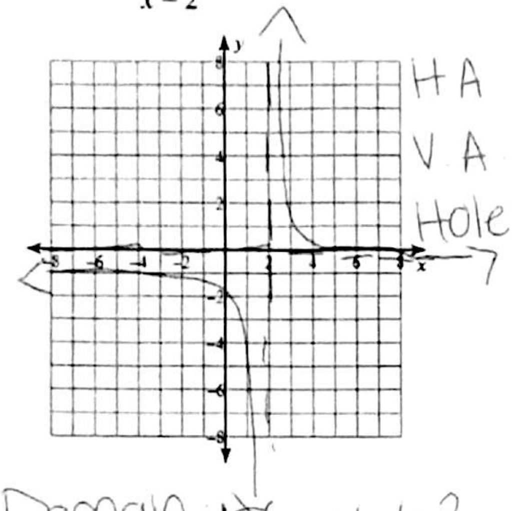
Graph the following examples using a graphing calculator.

Graph the asymptotes using dotted lines.

Graph the holes using an open dot.

State the domain and range.

1) $f(x) = \dfrac{2}{x-2}$



H A $y=0$

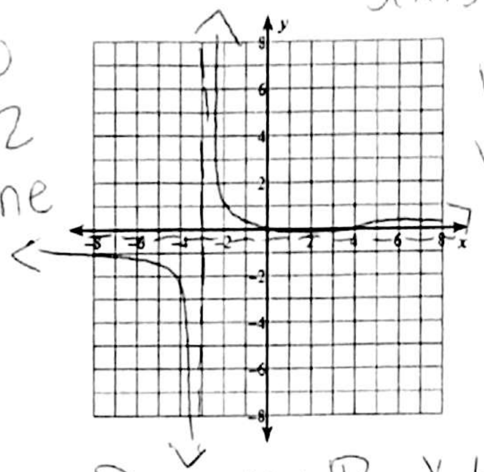
V A $x=2$

Hole none

Domain: $\mathbb{R}$, $x \neq 2$

Range: $\mathbb{R}$, $x \neq 1$

2) $f(x) = \dfrac{x-3}{-3x-9} = \dfrac{x-3}{-3(x+3)}$



H A $y = -\frac{1}{3}$

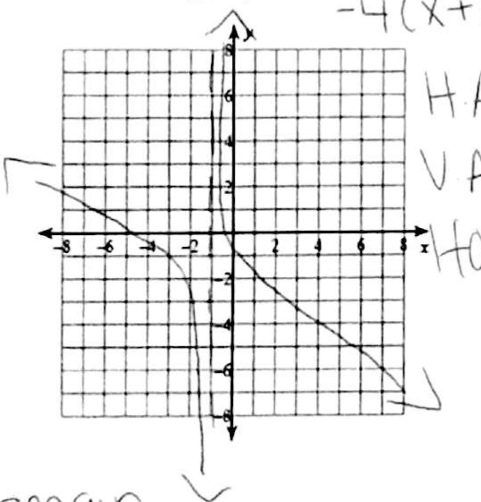
V A $x = -3$

Hole none

Domain: $\mathbb{R}$ $x \neq -3$

Range: $\mathbb{R}$ $y \neq -1/3$

3) $f(x) = \dfrac{x^2+x-2}{-4x-4} = \dfrac{(x-1)(x+2)}{-4(x+1)}$



H.A. none

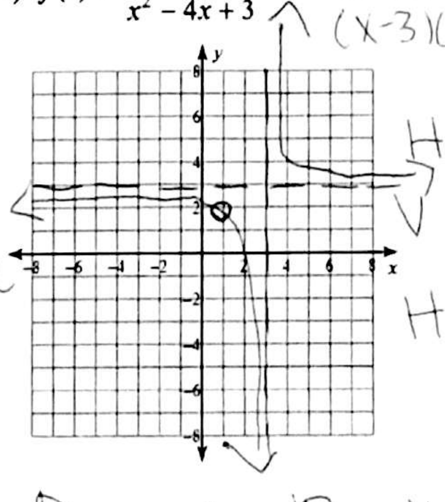
V.A. $x=-1$

Hole none

Domain: $\mathbb{R}$ $x \neq -1$

Range: $\mathbb{R}$

4) $f(x) = \dfrac{3x^2-9x+6}{x^2-4x+3} = \dfrac{3(x^2-3x+2)}{(x-3)(x-1)} = \dfrac{3(x-2)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$



H.A. $y=3$

V.A. $x=3$

Hole: $x=1$

Domain: $\mathbb{R}$, $x \neq 1, 3$

Range: $\mathbb{R}$, $y \neq 1.5, 3$