Unit 5: Solving Polynomials

Guided Notes

KEY

Name

Period

If found, please return to Mrs. Brandley's room, M-8.

Concept 1: Solving Polynomials by Factoring

<u>Factoring GCF</u>

Factor out the GCF and rewrite.

Examples: 1. $6x^3 - 8 = 0$ 2. $4x^2 + 12x = 0$ 3. $3x^2 - 9x = 0$

$2(3x^3-4)$

$4x(x+3)$

$\frac{4x}{4}=\frac{0}{4}$ $\boxed{x=0}$ $x+3=0$ $-3\ \ -3$ $\boxed{x=-3}$

$3x(x-3)$

$\frac{3x}{3}=\frac{0}{3}$ $\boxed{x=0}$ $x-3=0$ $+3\ \ +3$ $\boxed{x=3}$

<u>Factoring Quadratics when A=1</u>

1. Write the quadratic in standard form. $Ax^2 + Bx + C = 0$

2. Factor out the greatest common factor (GCF). (There will not always be one).

3. Find two numbers (we'll call them d and e) that multiply to AxC and add to B.

4. Rewrite in factored $(x + d)(x + e)$

Examples: 4. $x^2 + 5x + 6 = 0$ 5. $x^2 - 4x - 12 = 0$ 6. $x^2 - 6x + 8 = 0$

AxC=6 B=5
2 & 3
$(x+2)(x+3)$
$x=-2,-3$

AxC=-12 B=-4
-6 & 2
$(x-6)(x+2)$
$x=6,-2$

AxC=8 B=-6
-4 & -2
$(x-4)(x-2)$
$x=4,2$

<u>Factoring Quadratics when $A \neq 1$</u>

1. Write the quadratic in standard form. $Ax^2 + Bx + C = 0$

2. Factor out the greatest common factor (GCF). (There will not always be one).

3. Find two numbers that multiply to AxC and add to B.

4. Rewrite the quadratic splitting up the x-term into two terms using the two numbers found in step 3.

5. Split the quadratic into two groups and factor out the GCF of each side.

6. Factor out the term in parentheses and rewrite in factored form.

Examples: 7. $2x^2 + x - 6 = 0$ 8. $4x^2 - 19x + 12 = 0$ ~~9. $5x^2 - 10x + 6 = 0$~~

AxC=-12 B=1
-3 & 4
$2x^2+4x-3x-6$
$2x(x+2)-3(x+2)$
$(2x-3)(x+2)$
$x=-2, 3/2$

AxC=48 B=-19
-16 & -3
$4x^2-16x-3x+12$
$4x(x-4)-3(x-4)$
$(4x-3)(x-4)$
$x=3/4, 4$

~~AxC=30~~

Difference of Squares

Follow the same steps as when factoring quadratics with b=0.

Examples: 1. $x^2 - 25 = 0$

$x^2 + 0x - 25$

AC: -25 B: 0

-5 & 5

$(x-5)(x+5)$

$x = 5, -5$

2. $4x^2 - 9 = 0$

$4x^2 + 0x - 9$

AC: -36 B: 0

-6 & 6

$4x^2 - 6x + 6x - 9$

$2x(2x-3) + 3(2x-3)$

$(2x-3)(2x+3)$

$x = 3/2, -3/2$

3. $x^2 - 64 = 0$

$x^2 + 0x - 64$

-8 & 8

$(x-8)(x+8)$

$x = 8, -8$

Factoring Quadratic Type

Follow the same steps as given for quadratics except when rewriting it will be in the form $(x^2 + d)(x^2 + e)$.

Examples: 4. $x^4 + 5x^2 + 6 = 0$

A×C: 6 B: 5

2 & 3

$(x^2+2)(x^2+3)$

5. $x^4 - 4x^2 - 12 = 0$

A×C: -12 B: -4

-6 & 2

$(x^2-6)(x^2+2)$

~~6. $5x^4 - 10x^2 + 6 = 0$~~

$2x^4 + x^2 - 6$

A×C -12 B: 1

-3 & 4

$2x^4 - 3x^2 + 4x^2 - 6$

$x^2(2x^2-3) + 2(2x^2-3)$

$(2x^2-3)(x^2+2)$

Factoring by Grouping

1. Split the quadratic into two groups and factor out the GCF of each side.

2. Factor out the term in parentheses and rewrite in factored form.

Examples:

7. $(x^3 + 7x^2) + (2x + 14) = 0$

$x^2(x+7) + 2(x+7)$

$(x+7)(x^2+2)$

$x + 7 = 0$ $\quad$ $x^2 + 2 = 0$

$\boxed{x = -7}$ $\quad$ $\sqrt{x^2} = \sqrt{-2}$

$\boxed{x = \pm i\sqrt{2}}$

8. $(x^3 - 2x^2) + (5x - 10) = 0$

$x^2(x-2) + 5(x-2)$

$(x-2)(x^2+5)$

$x - 2 = 0$ $\quad$ $x^2 + 5 = 0$

$\boxed{x = 2}$ $\quad$ $\sqrt{x^2} = \sqrt{-5}$

$\boxed{x = \pm i\sqrt{5}}$

9. $(x^3 + x^2)(-4x - 4) = 0$

$x^2(x+1) - 4(x+1)$

$(x+1)(x^2-4)$

$x + 1 = 0$ $\quad$ $x^2 - 4 = 0$

$\boxed{x = -1}$ $\quad$ $\sqrt{x^2} = \sqrt{4}$

$\boxed{x = \pm 2}$

Factoring Using Sum/Difference of Cubes

Rewrite the constant number as a number cubed, then use the following formulas:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples:

$x^3 + 64 = x^3 + 4^3$ $a = x$ $b = 4$

$(x+4)(x^2 - 4x + 16)$

$x^3 - 125 = x^3 - 5^3$ $a = x$ $b = 5$

$(x-5)(x^2 + 5x + 25)$

What is the difference between **factoring** and **solving**?

Factoring: write in factored form, (ex $(x+5)(x-2)$)

Solving: Factor then set each factor equal to 0 and solve for x, (ex: $x = -5, 2$)

How do I know which type of factoring to use?

GCF: ALWAYS DO THIS FIRST!! If there is no GCF other than 1, then look for another factoring method. If there is a GCF, factor it out then see if you can factor it further or if it is as simplified as possible.

Factoring an a=1 quadratic: If the highest exponent is a 2 and, after pulling out the GCF if, a=1 meaning there is no number in front of the x^2.

Factoring an $a \neq 1$quadratic: If the highest exponent is a 2 and after pulling out the GCF (if there is one), a is not 1 meaning there is a number in front of the x^2.

Difference of Squares: If the highest exponent is a 2 and there is no x term, only an x^2 term and a constant term. It also has to be subtracting, not adding, in order to be factorable.

Factoring Quadratic Type: If it is written in the form $Ax^4 + Bx^2 + C$, there is only $x^4 terms, x^2 terms, and\ constant\ terms.$

Factor by Grouping: Try factoring by grouping if the polynomial has an exponent higher than 2 and is not a sum or difference of cubes.

Sum/Difference of Cubes: If it is of the form $x^3 + c$, or $x^3 - c$ meaning it is just an x^3 term and a constant.

Concept 2: Synthetic and Polynomial Long Division

Synthetic Division

1. $2x^4 - 5x^3 - 14x^2 + 47x - 30 \div (x - 2)$

the coefficients of f(x)

k from $x - k$

$$
\begin{array}{r|rrrrr}
2 & 2 & -5 & -14 & 47 & -30 \\
 & \downarrow & 4 & -2 & -32 & 30 \\
\hline
 & 2 & -1 & -16 & 15 & 0
\end{array}
$$

$2x^3 - 1x^2 - 16x + 15$

2. $x^4 - 3x^3 - 11x^2 + 5x + 17 \div (x + 2)$ ~~of 5~~

$$
\begin{array}{r|rrrrr}
-2 & 1 & -3 & -11 & 5 & 17 \\
 & \downarrow & -2 & 10 & 2 & -14 \\
\hline
 & 1 & -5 & -1 & 7 & 3
\end{array}
$$

$x^3 - 5x^2 - x + 7 + \frac{3}{x+2}$

3. $2x^3 - 13x^2 + 17x - 10$ with a factor of 5

$$
\begin{array}{r|rrrr}
5 & 2 & -13 & 17 & -10 \\
 & \downarrow & 10 & -15 & 10 \\
\hline
 & 2 & -3 & 2 & 0
\end{array}
$$

$2x^2 - 3x + 2$

Polynomial Long Division

1. $3x^3 + 2x - 11 \div (x - 3)$

$3x^3 + 0x^2 + 2x - 11 \div (x - 3)$

$$
\begin{array}{r}
3x^2 + 9x + 29 \\
x - 3 \,\overline{)\, 3x^3 + 0x^2 + 2x - 11} \\
\underline{-(3x^3 - 9x^2)} \qquad\qquad \\
9x^2 + 2x \qquad \\
\underline{-(9x^2 - 27x)} \qquad \\
29x - 11 \\
\underline{-(29x - 87)} \\
76
\end{array}
$$

2. $4x^3 + 2x^2 - 6x + 3 \div (x - 3)$

$$
\begin{array}{r}
4x^2 + 14x + 36 + \frac{111}{x-3} \\
x - 3 \,\overline{)\, 4x^3 + 2x^2 - 6x + 3} \\
\underline{-(4x^3 - 12x^2)} \qquad\qquad \\
14x^2 - 6x \qquad \\
\underline{-(14x^2 - 42x)} \qquad \\
+36x + 3 \\
\underline{-(36x - 108)} \\
111
\end{array}
$$

3. $14x^4 - 5x^3 - 11x^2 - 11x - 8 \div (2x - 1)$

$$
\begin{array}{r}
7x^3 + x^2 - 5x - 8 - \frac{16}{2x-1} \\
2x - 1 \,\overline{)\, 14x^4 - 5x^3 - 11x^2 - 11x - 8} \\
\underline{-(14x^4 - 7x^3)} \qquad\qquad\qquad \\
2x^3 - 11x^2 \qquad\qquad \\
\underline{-(2x^3 - x^2)} \qquad\qquad \\
-10x^2 - 11x \qquad \\
\underline{-(-10x^2 + 5x)} \qquad \\
-16x - 8 \\
\underline{-(-16x + 8)} \\
-16
\end{array}
$$

When can you use synthetic division instead of polynomial long division?

When the factor is a whole number.

EX: $x - 2$

$x = 2$

NON-EX: $2x - 1$

$x = 1/2$

Concept 3: Solving Polynomials using Division

Factor and solve the following. Use synthetic division to help you.

$x^3 + 3x^2 - 4$ with a factor of 1

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & \downarrow & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

x^2+4x+4

$(x+2)(x+2)$

$x=-2,-2,1$

$2x^4 + 3x^3 + 2x^2 + 6x - 4$ with a factor of -2

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & 2 & 6 & -4 \\ & \downarrow & -4 & 2 & -8 & 4 \\ \hline & 2 & -1 & 4 & -2 & 0 \end{array}$$

$(2x^3-x^2)+(4x-2)$

$x^2(2x-1)+2(2x-1)$

$(2x-1)(x^2+2)$

$x=1/2, \pm i\sqrt{2}, -2$

Factor and solve the following. Use polynomial long division to help you.

$x^3 + 3x^2 - 4 \div (x - 1)$

$$\begin{array}{l} \quad\quad\quad x^2+4x+4 \\ x-1 \overline{)\, x^3+3x^2+0x-4} \\ \quad -(x^3-x^2) \\ \hline \quad\quad 0 \;\; 4x^2+0x \\ \quad\quad -(4x^2-4x) \\ \hline \quad\quad\quad 0 \;\; 4x-4 \\ \quad\quad\quad -(4x-4) \\ \hline \quad\quad\quad\quad 0 \;\; 0 \end{array}$$

x^2+4x+4

$(x+2)(x+2)(x-1)$

$x=-2,-2,1$

$2x^4 + 3x^3 + 2x^2 + 6x - 4 \div (x + 2)$

$$\begin{array}{l} \quad\quad\quad 2x^3-x^2+4x-2 \\ x+2 \overline{)\, 2x^4+3x^3+2x^2+6x-4} \\ \quad -(2x^4+4x^3) \\ \hline \quad\quad -x^3+2x^2 \\ \quad\quad -(-x^3-2x^2) \\ \hline \quad\quad\quad 4x^2+6x \\ \quad\quad\quad -(4x^2+8x) \\ \hline \quad\quad\quad\quad -2x-4 \\ \quad\quad\quad\quad -(-2x-4) \\ \hline \quad\quad\quad\quad\quad 0 \end{array}$$

$(2x^3-x^2)+(4x-2)$

$x^2(2x-1)+2(2x-1)$

$(x^2+2)(2x-1)(x+2)$

$x=\pm i\sqrt{2}, 1/2, -2$