Unit 1: Exponents and Radicals
Guided Notes

KEY
Name

Period

If found, please return to Mrs. Brandley's room, M-8.

Self-Assessment

The following are the concepts you should know by the end of Unit 1. Periodically throughout the unit I will ask you to self-assess on how you are doing on these skills. It is essential for you to be able to identify what you do and do not understand in order to learn effectively. You will use the following scale:

5: Yes! I understand
4: I'm almost there.
3: I am back and forth.
2: I am just starting to understand.
1: I don't understand at all.

Concept 1: Order of Operations

______ I know what the order of operations is and how it is used.

______ I can use the order of operations to solve expressions with real numbers.

Concept 2: Types of Numbers

______ I know what a complex number is.

______ I can simplify square roots of negative numbers.

______ I can identify what type of number (e.g. rational, integer, etc.) a given number is.

______ I know whether the sums or products of given numbers are rational or irrational.

Concept 3: Understanding Exponents and Radicals

______ I know what the product property of exponents is and how to use it.

______ I know what the quotient property of exponents is and how to use it.

______ I know what the power property of exponents is and how to use it.

______ I know what the zero exponent property is and how to use it.

______ I know what the negative exponent property is and how to use it.

Concept 4: Going Between Exponent and Radical Form

______ I can switch back and forth between radical and exponent form.

______ I can simplify radicals using prime factorization.

Concept 5: Simplifying Rational Exponents

______ I know what it means for an expression to be in "simplest form".

______ I can simplify expressions involving radicals and/or rational exponents.

Concept 1: Order of Operations

1. Two people solve the following problem in the two different ways shown. Which do you think is correct, and why?

Person A	Person B
$8 - 2 + 1$	$8 - 2 + 1$
$6 + 1$	$8 - 3$
7	5

Person A. The order of operations does addition and subtraction from left to right

2. The same two people solve the following problem in the two different ways shown. Which do you think is correct, and why?

Person A	Person B
$6 - 3 \times 3$	$6 - 3 \times 3$
3×3	$6 - 9$
9	-3

Person B. The order of operations does Multiplication before subtraction

3. What is the order of operations?

P: Parentheses or other grouping symbols

E: Exponents

→MD: Multiplication & Division from left to right

→AS: Addition and Subtraction from left to right

4. What is your favorite number?

5. On the index card given to you, write an expression that is equal to your favorite number and includes at least 4 of the following:
   a. Parentheses (or other grouping symbol)
   b. Exponents
   c. Multiplication
   d. Division
   e. Addition
   f. Subtraction

6. EXAMPLE: 72

   72

   9×8

   $(7+2) \times 8$

   $(7+2) \times (10-2)$

7. Trade index cards with the person next to you. Try to solve their problem. When you are both finished, compare your answer to their favorite number. Are they the same? If not, solve the problem together and see if you can come to agreement on what the answer should be. If a mistake was made on either side, that's great! That is the best way to make your brain grow!

8. Solve your problem here:

9. Solve your partner's problem here:

Concept 2: Types of Numbers

Fill in the word for each of the following definitions:

Natural Numbers the counting numbers. {1, 2, 3 4, 5, 6, 7, 8, 9, ...}

EX: 321

Whole Numbers the natural numbers including 0 {0, 1, 2, 3, 4, 5, 6...}

EX: 0, 756

Integers the whole numbers plus their opposites {....-3, -2, -1, 0, 1, 2, 3....}

EX: -723

Rational Numbers all the integers and all numbers that can be written as a fraction of integers, which are terminating or repeating decimals

EX: 3/4

irrational Numbers all real numbers that can't be written as a fraction of two integers, decimals that don't repeat or terminate

EX: $\sqrt{2}$

Real Numbers all rational and irrational numbers

EX: all rational and irrational numbers

Imaginary Numbers square roots of negative numbers such that $i^2 = -1$ and $i = \sqrt{-1}$.

EX: $7i$

Complex Numbers combinations of imaginary numbers and real numbers

EX: $5i + 6$

What happens when you add two rational numbers? Is the result always another rational number, or can it be irrational? Will the sum of two irrational numbers always be rational, always be irrational, or can it be either? To find out, with your group, fill in the following table:

+	-2π	9	$\frac{1}{2}$	0	$\sqrt{5}$	$-\sqrt{5}$
-2π	-4π	$9-2\pi$	$\frac{1}{2}-2\pi$	-2π	$\sqrt{5}-2\pi$	$-\sqrt{5}-2\pi$
9	$9-2\pi$	18	$9\frac{1}{2}$	9	$\sqrt{5}+9$	$-\sqrt{5}+9$
$\frac{1}{2}$	$\frac{1}{2}-2\pi$	$9\frac{1}{2}$	1	$1/2$	$\sqrt{5}+\frac{1}{2}$	$-\sqrt{5}+\frac{1}{2}$
0	2π	9	$\frac{1}{2}$	0	$\sqrt{5}$	$-\sqrt{5}$
$\sqrt{5}$	$\sqrt{5}-2\pi$	$\sqrt{5}+9$	$\sqrt{5}+\frac{1}{2}$	$\sqrt{5}$	$2\sqrt{5}$	0
$-\sqrt{5}$	$-\sqrt{5}-2\pi$	$-\sqrt{5}+9$	$-\sqrt{5}+\frac{1}{2}$	$-\sqrt{5}$	0	$-2\sqrt{5}$

Based on the results in the table, complete the following statements:

The sum of two rational numbers will ______________ be a rational number.

A. Always

B. Sometimes

C. Never

The sum of two irrational numbers will ______________ be a rational number.

A. Always

B. Sometimes

C. Never

The sum of a rational and irrational number will ______________ be a rational number

A. Always

B. Sometimes

C. Never

What about the product of rational and irrational numbers? Complete the following table:

$\times$	-2π	9	$\frac{1}{2}$	0	$\sqrt{5}$	$-\sqrt{5}$
-2π	$4\pi^2$	-18π	$-\pi$	0	$-2\pi\sqrt{5}$	$2\pi\sqrt{5}$
9	-18π	81	$\frac{9}{2}$	0	$9\sqrt{5}$	$-9\sqrt{5}$
$\frac{1}{2}$	$-\pi$	$9/2$	$\frac{1}{4}$	0	$\sqrt{\frac{5}{2}}$	$\frac{-\sqrt{5}}{2}$
0	0	0	0	0	0	0
$\sqrt{5}$	$-2\pi\sqrt{5}$	$9\sqrt{5}$	$\frac{\sqrt{5}}{2}$	0	5	-5
$-\sqrt{5}$	$2\pi\sqrt{5}$	$-9\sqrt{5}$	$-\frac{\sqrt{5}}{2}$	0	-5	5

Based on the results in the table, complete the following statements:

The product of two rational numbers will _______________ be a rational number.

A. Always

B. Sometimes

C. Never

The product of two irrational numbers will _______________ be a rational number.

A. Always

B. Sometimes

C. Never

The product of a rational and irrational number will ______________ be a rational number

A. Always

B. Sometimes

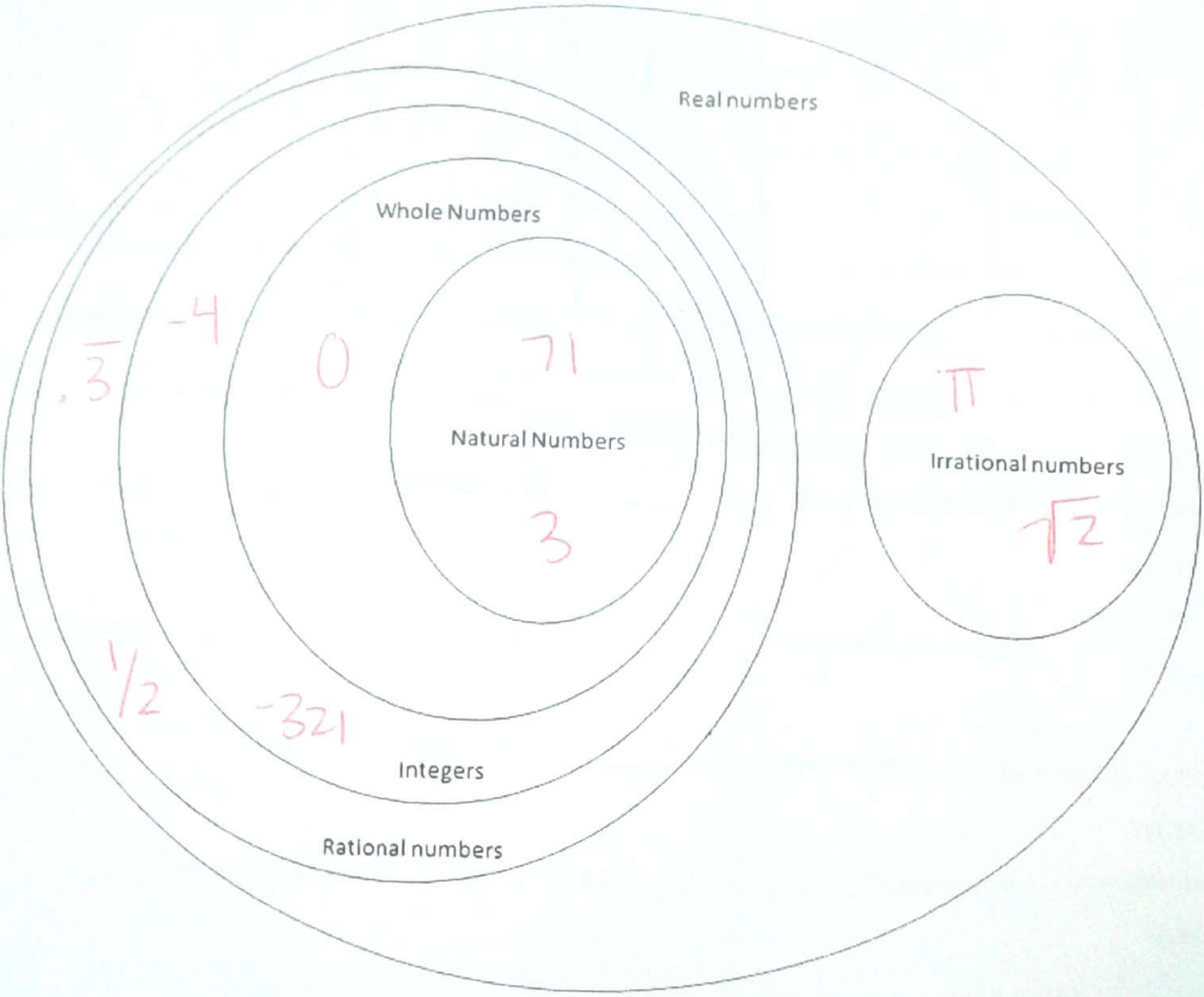
C. Never

The product of a **nonzero** rational number and an irrational number will____________be a rational number.

A. Always

B. Sometimes

C. Never



Concept 3: Understanding Exponents and Radicals

Product Property

A: For each expression, evaluate and explain how you got your answer:

$2^2 =$ 4 $2 \times 2 = 4$

$3^2 =$ 9 $3 \times 3 = 9$

$2^3 =$ 8 $2 \times 2 \times 2 = 8$

B: Write these in an "expanded" form. Do not find a solution, just rewrite them.

3^4 $3 \times 3 \times 3 \times 3$

4^2 4×4

x^7 $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

C: Write these as an expression with one exponent. Once again, do not find the solution.

$x \cdot x \cdot x$ x^3

$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ 8^5

3×3 3^2

$(7)(7)(7)(7)$ 7^4

D: "Expand" each exponential expression:

$2^2 \cdot 2^3$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$(4^3)(4^2)$ $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$a^1 \times a^4$ $a \cdot a \cdot a \cdot a \cdot a$

$5^3 \cdot 5^1$ $5 \cdot 5 \cdot 5 \cdot 5$

E. Now, with the expressions you just "expanded", rewrite these expressions with only one exponent.

1. 2^6 2. 4^5 3. a^5 4. 5^4

F: With your partner, rewrite the following as an expression with only one exponent.

x^2x^3 x^5

$4^{12}4^9$ 4^{21}

G: Also with your partner, write down an algorithm (a generalized formula or process to follow) that you used to rewrite the expressions above.

$x^a x^b = x^{a+b}$

Quotient Property

A. Simplify the following and explain how you know:

$\frac{x}{x} = 1$. Any number divided by itself is equal to 1

B. Expand the numerators and denominators of the following, simplify each x/x to be 1 and then write as an expression with just one exponent (see example):

EX: $\frac{x^6}{x^2} = \frac{xxxxxx}{xx} = \frac{x}{x} \times \frac{x}{x} \times xxxx = 1 \times 1 \times xxxx = xxxx = x^4$

$\frac{x^5}{x^3} = \frac{xxxxx}{xxx} = \frac{x}{x} \cdot \frac{x}{x} \cdot \frac{x}{x} \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot x \cdot x = x^2$

$\frac{x^9}{x^2} = \frac{xxxxxxxxx}{xx} = \frac{x}{x} \cdot \frac{x}{x} \cdot xxxxxxx = 1 \cdot 1 \cdot xxxxxxx = x^7$

$\frac{x^3}{x^5} = \frac{xxx}{xxxxx} = \frac{x}{x} \cdot \frac{x}{x} \cdot \frac{x}{x} \cdot \frac{1}{xx} = 1 \cdot 1 \cdot 1 \cdot \frac{1}{xx} = \frac{1}{x^2}$

C. With your partner, write down an algorithm (a generalized formula or process to follow) that you used to rewrite the expressions above.

$$\frac{x^a}{x^b} = x^{a-b}$$

Power Property

A. Expand and then write as an expression with just one exponent (see example):

EX: $(x^4)^2 = x^4x^4 = xxxxxxxx = x^8$

$(x^3)^4 = x^3x^3x^3x^3 = xxxxxxxxxxxx = x^{12}$

$(x^2)^5 = x^2x^2x^2x^2x^2 = xxxxxxxxxx = x^{10}$

$(x^5)^2 = x^5x^5 = xxxxxxxxxx = x^{10}$

C. With your partner, write down an algorithm (a generalized formula or process to follow) that you used to rewrite the expressions above.

$$(x^a)^b = x^{ab}$$

Zero Exponent Property

A. Expand the numerator of each of the following, and then simplify to x to a single exponent as seen in the first example:

$$\frac{x^4}{x} = \frac{xxxx}{x} = xxx = x^3$$

$$\frac{x^3}{x} = \frac{xxx}{x} = xx = x^2$$

$$\frac{x^2}{x} = \frac{xx}{x} = x^1 = x$$

$$\frac{x^1}{x} = x^1 = x$$

B. Evaluate the following:

$x^0 = 1$ $\quad 4^0 = 1$ $\quad (-2)^0 = 1$ $\quad (\frac{2}{3})^0 = 1$

C. With your partner, write down an algorithm (a generalized formula or process to follow) that you used to rewrite the expressions above.

$X^0 = 1$

Negative Exponents Property

We noticed in the above experiments that each time we decreased the exponent by 1, we divided by x. Thus, if we continue this pattern we will move into negative exponents:

$$\frac{x^2}{x} = x^1$$

$$\frac{x^1}{x} = x^0 = 1$$

$$\frac{1}{x} = x^{-1}$$

$$\frac{1}{x^2} = x^{-2}$$

$$\frac{1}{x^3} = x^{-3}$$

Rewrite the following:

$x^{-7} = \frac{1}{X^7}$ $\quad \frac{1}{x^3} = X^{-3}$ $\quad 4^{-2} \; \frac{1}{4^2} = \frac{1}{16}$

C. With your partner, write down an algorithm (a generalized formula or process to follow) that you used to rewrite the expressions above.

$X^{-a} = \frac{1}{X^a}$

Properties of Exponents

Write the generalized formula for each of the properties below:

Product Property: $X^a X^b = X^{a+b}$

Quotient Property: $\frac{X^a}{X^b} = X^{a-b}$

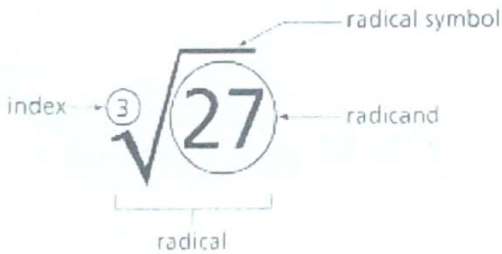
Power Property: $(X^a)^b = X^{ab}$

Zero Exponent Property: $X^0 = -1$

Negative Exponent Property: $X^{-a} = \frac{1}{X^a}$

As we are working with exponents and radicals, use the following diagram to remember the names of each part:

Parts of a Radical



Concept 4: Going Between Exponent and Radical Forms

In order to make our lives easier, we are going to assume that fractional exponents (known as rational exponents) follow the same properties as integer exponents like the ones above. If this is the case, solve the following:

$(3^{1/2})^2$ $3^1 = 3$

$4^{1/2}4^{1/2}$ $4^1 = 4$

Now solve the following:

$\sqrt{3}^2$ $= 3$

$\sqrt{4} \times \sqrt{4}$ $= 4$

You should have gotten the same answer (3) for $(3^{1/2})^2$ and $\sqrt{3}^2$ and the same answer again (4) for $4^{1/2}4^{1/2}$ and $\sqrt{4} \times \sqrt{4}$.

But this means that $3^{1/2}$ is the same as $\sqrt{3}$ and $4^{1/2}$ is the same as $\sqrt{4}$.

In order for the exponent properties to hold true for rational exponents, the following must be true:

$$x^{a/b} = \sqrt[b]{x^a}$$

Remember that when there is no index, the index is 2. (i.e. $\sqrt{x} = \sqrt[2]{x}$)

It can often come in handy to switch between exponent and radical form in order to make problems easier to solve. Try switching forms for the following:

As a class…

$x^{a/b}$ $\sqrt[b]{x^a}$ $(3x)^{4/5}$ $\sqrt[5]{(3x)^4}$ $\sqrt[2]{x^4}$ $x^{\frac{4}{2}} = x^2$ $2x^{4/2}$ $2\sqrt{x^4}$

As a group…

$4x^{3/7}$ $4\sqrt[7]{x^3}$ $x^{3/4}$ $\sqrt[4]{x^3}$ $\sqrt[5]{x^4}$ $x^{4/5}$ $(5x)^{7/2}$ $\sqrt{(5x)^7}$

With your partner…

$x^{4/5}$ $\sqrt[5]{x^4}$ $2x^{1/4}$ $2\sqrt[4]{x}$ $\sqrt[5]{x^3}$ $x^{3/5}$ $(9x)^{4/9}$ $\sqrt[9]{(9x)^4}$

By yourself…

$x^{1/5}$ $\sqrt[5]{x}$ $7x^{4/7}$ $7\sqrt[7]{x^4}$ $\sqrt[9]{x^4}$ $x^{4/9}$ $(8x)^{5/2}$ $\sqrt{(8x)^5}$

Simplifying Radicals Review

It is much easier to simplify radicals when we break the number down into it's prime factorization:

What does prime mean? A number is prime if its' only factors are 1 and itself.

Try simplifying the following radicals using prime factorization:

$\sqrt{8}$

4 2

2 2

$\sqrt{2 \cdot 2 \cdot 2}$

$\boxed{2\sqrt{2}}$

$\sqrt[3]{8}$

4 2

2 2

$\sqrt{2 \cdot 2 \cdot 2}$

$\boxed{2}$

$\sqrt{60}$

10 6

5 2 2 3

$\sqrt{5 \cdot 2 \cdot 2 \cdot 3}$

$2\sqrt{5 \cdot 3}$

$\boxed{2\sqrt{15}}$

$\sqrt[3]{108}$

54 2

9 6

3 3 3 2

$\sqrt[3]{3 \cdot 3 \cdot 3 \cdot 2 \cdot 2}$

$3\sqrt[3]{2 \cdot 2}$

$\boxed{3\sqrt[3]{4}}$

Concept 5: Simplifying Rational Exponents

You have now learned everything you need to know in order to be able to simplify rational exponents we just need to put it all together!

Note that *"simplest form"* means *"radical form and that all possible factors have been taken into consideration and all remaining factors have been multiplied."*

As a class…

$\sqrt{12n}$ $\quad 2\sqrt{3n}$

$\sqrt{48n^2}$ $\quad 4n\sqrt{3}$

$(64p^6)^{1/2}$ $\quad 8p^3$

$(81x^6)^{3/2}$ $\quad \sqrt{(81x^6)^3} = \sqrt{81^3 x^{18}}$ $\quad 81x^9\sqrt{81} = 81\cdot 9x^9$ $\quad \boxed{729x^9}$

As a group…

$\sqrt{18a^2}$ $\quad 3a\sqrt{2}$

$(n^9)^{5/3}$ $\quad n^{45/3} = \boxed{n^{15}}$

$(81x^4)^{3/2}$ $\quad 81^{3/2}x^6$ $\quad \boxed{729x^6}$

With your partner…

$\sqrt{80v^2}$ $\quad 4v\sqrt{5}$

$(n^{12})^{4/3}$ $\quad n^{48/3} = \boxed{n^{16}}$

$(16v^2)^{3/2}$ $\quad 16^{3/2}v^3$ $\quad \boxed{64v^3}$

By yourself...

$\sqrt{100b}$ $\quad 10\sqrt{b}$

$(81k^4)^{1/2}$ $\quad \sqrt{81}k^2$ $\quad \boxed{9k^2}$

$(r^4)^{3/2}$ $\quad r^{12/2}$ $\quad \boxed{r^6}$