

Unit 3: Functions  
Guided Notes

KEY

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Name

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Period

\*\*If found, please return to Mrs. Brandley's room, M-8.\*\*

## Self-Assessment

The following are the concepts you should know by the end of Unit 1. Periodically throughout the unit I will ask you to self-assess on how you are doing on these skills. It is essential for you to be able to identify what you do and do not understand in order to learn effectively. You will use the following scale:

- 5: Yes! I understand
- 4: I'm almost there.
- 3: I am back and forth.
- 2: I am just starting to understand.
- 1: I don't understand at all.

### Concept 1: Defining Functions

- \_\_\_\_\_ I understand that a function is a relationship where every input has a unique output.
- \_\_\_\_\_ I can identify whether or not a graph does or does not represent a function.
- \_\_\_\_\_ I can identify whether or not a table does or does not represent a function.
- \_\_\_\_\_ I can identify whether or not an equation does or does not represent a function.

### Concept 2: Domain and Range

- \_\_\_\_\_ I can define domain and range.
- \_\_\_\_\_ I can identify the domain and range of a function based on its' graph.
- \_\_\_\_\_ I can identify domain and range of a function based on its' mapping, table, or set of ordered pairs.
- \_\_\_\_\_ I can identify the domain of a function based on its' equation.

### Concept 3: Writing and Evaluating Functions

- \_\_\_\_\_ I can evaluate a function given a specific input value.
- \_\_\_\_\_ I can add, subtract, and multiply functions.
- \_\_\_\_\_ I can write a linear equation for a function based on a table.
- \_\_\_\_\_ I can write a quadratic equation for a function based on a table.

### Concept 4: Average Rate of Change

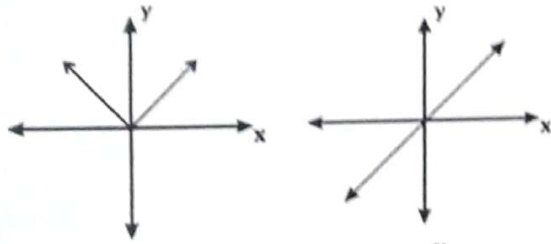
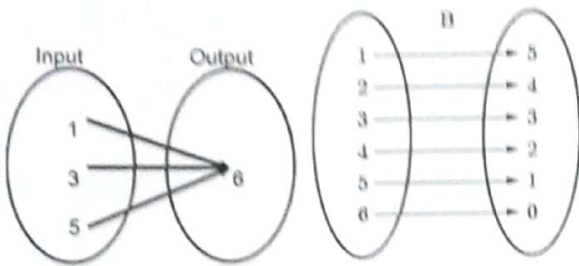
- \_\_\_\_\_ I can find the average rate of change given an equation for a function and an interval.
- \_\_\_\_\_ I can find the average rate of change given a table and an interval.
- \_\_\_\_\_ I can estimate the average rate of change given a graph and an interval.

### Concept 1: Defining Functions

Using the following examples and non-examples of functions, on the next page, write down characteristics of a function. When you are finished, discuss with your group and write a definition of a function.

#### FUNCTION

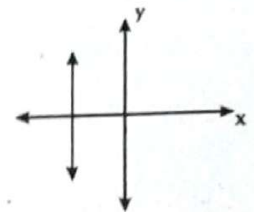
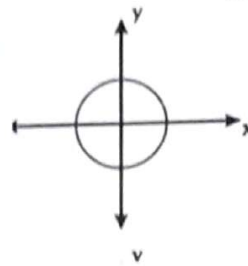
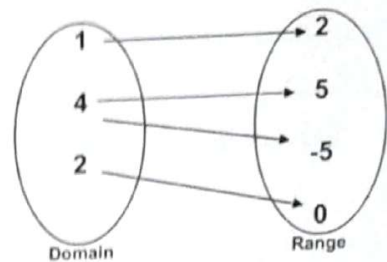
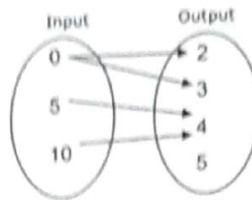
##### EXAMPLES



x	-1	2	5
y	-4	8	20

x	f(x)
0	0
1	1
4	2
9	3
16	4

##### NON-EXAMPLES



x	y
3	6
4	6
5	7
5	8
6	10
10	9
11	11

x	1	2	4	4	7	9	25
y	6	5	4	3	2	0	-5

Characteristics of a Function: there's an x and y, there can only be one output for each input but there can be multiple inputs for one output, graphs pass the vertical line test

Your Group's Definition: \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

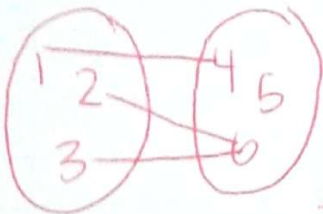
The Class Definition: Function: a relationship between x (input) and y (output) where every input has exactly one output.

Based on the definition we decided on as a class, write one example and one non-example of a function in each of the following forms:

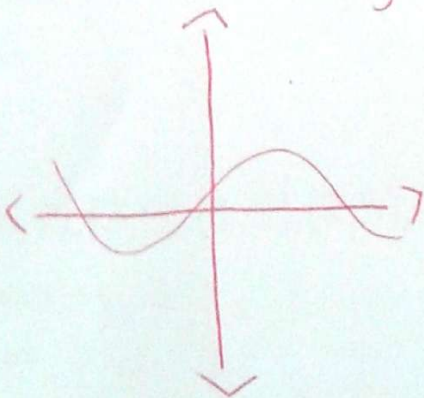
- Mapping
- Table
- Graph

**FUNCTION**

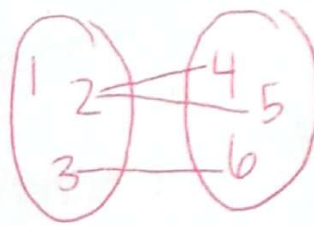
**EXAMPLES**



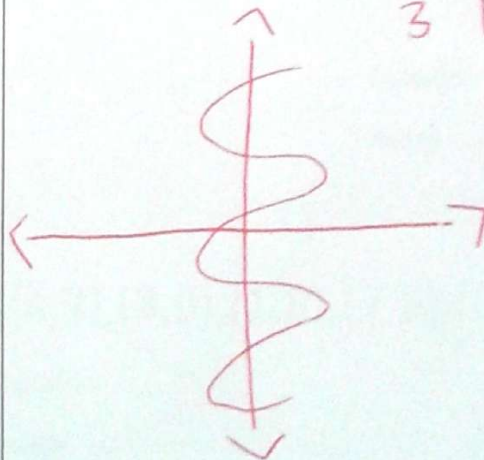
X	Y
1	4
2	5
3	6
3	6



**NON-EXAMPLES**



X	Y
1	4
2	5
3	6
3	7

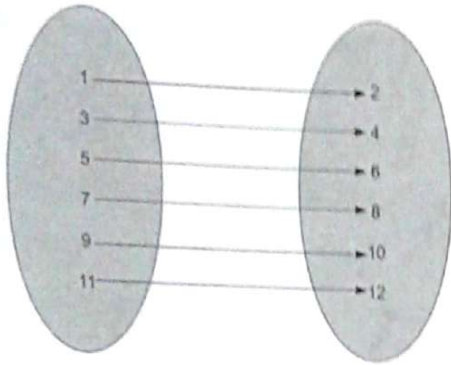


### Concept 2: Domain and Range of Functions

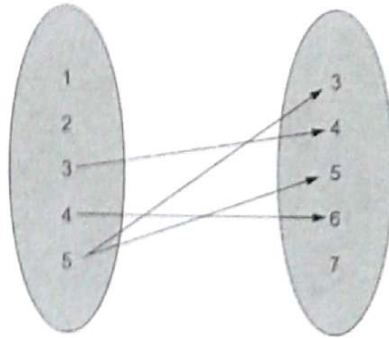
Domain: The set of all possible input values (x-values) of a function.

Range: The set of all possible output values (y-values) of a function.

State the domain and range of the following:



Domain: 1, 3, 5, 7, 9, 11  
Range: 2, 4, 6, 8, 10, 12

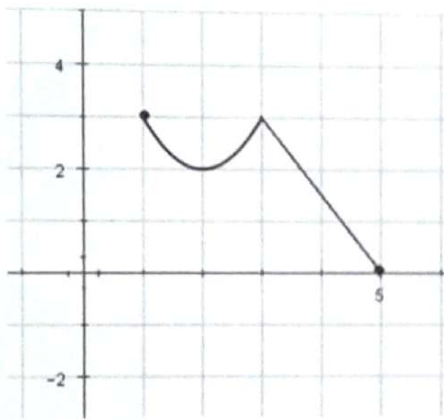


Domain: 1, 2, 3, 4, 5  
Range: 3, 4, 5, 6, 7

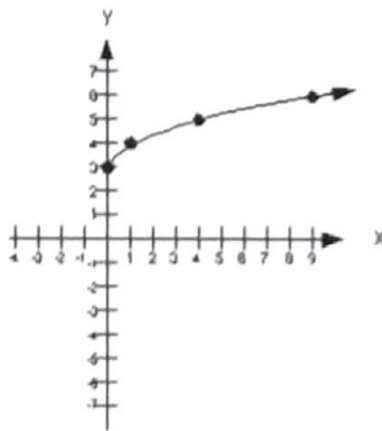
x	y
1	1
2	4
3	9
4	16
5	25

Domain: 1, 2, 3, 4, 5  
Range: 1, 4, 9, 16, 25

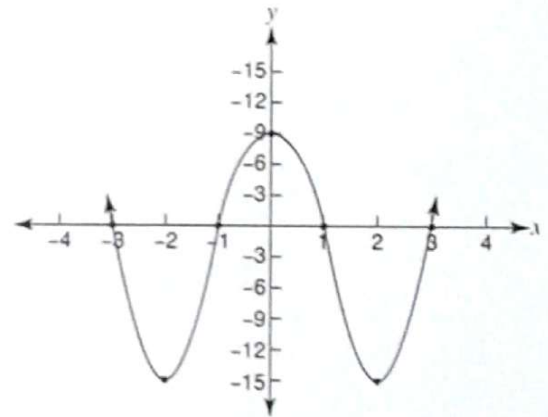
State the domain and range of the following graphs:



Domain:  $[1, 5]$   
Range:  $[0, 3]$



Domain:  $[0, \infty)$   
Range:  $[3, \infty)$



Domain:  $(-\infty, \infty)$   
Range:  $[-15, \infty)$

$\{(4, 7), (3, 9), (1, 8), (7, 5), (6, 2)\}$

Domain: 4, 3, 1, 7, 6  
Range: 7, 9, 8, 5, 2

~~$\{(4, 7), (3, 9), (1, 8), (7, 5), (6, 2)\}$~~

~~Domain:  
Range:~~

How do you find the domain when you are only given the algebraic representation of a function?

The only numbers you don't want to include in the domain are numbers that would result in a non-real output. What situations can you think of that give you solutions that aren't real numbers?

1. square roots of negative numbers
2. dividing by zero

Find the domain of the following functions:

$$f(x) = x^2 + 3$$

$$(-\infty, \infty)$$

$$f(x) = \frac{3-x}{x-2}$$

$$(-\infty, 2) \cup (2, \infty)$$

$$f(x) = \sqrt{x+5}$$

$$[-5, \infty)$$

Give two examples of when you might want to restrict the domain of a function:

1. when  $x$  represents something tangible
2. when  $x$  represents something finite

### Concept 3: Writing and Evaluating Functions

Function Notation

$$f(x) = y$$

Evaluating a function at the given values:

$$f(x) = x^2$$

$$f(2) = 2^2 = 4$$

$$g(x) = x - 5$$

$$g(9) = 9 - 5 = 4$$

$$h(x) = x^2 + 2$$

$$f(x) + h(x)$$

$$(x^2) + (x^2 + 2) = 2x^2 + 2$$

$$h(x) - g(x)$$

$$(x^2 + 2) - (x - 5) = x^2 - x + 7$$

$$f(x) * g(x)$$

$$x^2(x-5) = x^3 - 5x^2$$

$$f(g(x))$$

$$(x-5)^2 = (x-5)(x-5) = x^2 - 10x + 25$$

How do you know if a given table represents a linear function or a quadratic function?

Linear: if the differences in  $y$ -values remain constant.

Quadratic: if the second differences in  $y$ -values remain constant

1

x	y
-6	42
-5	34
-4	26
-3	18
-2	10

$> 8$   
 $> 8$   
 $> 8$   
 $> 8$   
 $> 8$

2

x	y
-6	-216
-5	-150
-4	-96
-3	-54
-2	-24

$> 66 > 12$   
 $> 54 > 12$   
 $> 42 > 12$   
 $> 30$

3

x	y
-6	72
-5	50
-4	32
-3	18
-2	8

$> 22 > 4$   
 $> 18 > 4$   
 $> 14 > 4$   
 $> 10$

4

x	y
0	8
1	6
2	4
3	2
4	0

$> 2$   
 $> 2$   
 $> 2$   
 $> 2$

Below, indicate if the previous tables represent linear or quadratic functions:

1. Linear
2. Quadratic
3. Quadratic
4. Linear

How to write a Linear Function from a table:

1. Pick two points.
2. Find the slope of the line  $\frac{y_1 - y_2}{x_1 - x_2}$
3. Plug one of the points and the slope into point-slope form.  $y - y_1 = m(x - x_1)$
4. Simplify the equation to  $y = mx + b$

EX: (6,5) and (2,4)

$$\text{EX: } \frac{5-4}{6-2} = \frac{1}{4}$$

EX:  $y - 5 = 1/4(x - 6)$

$$\text{EX: } y = \frac{x}{4} + 3.5$$

Write functions that represent the above two linear tables:

$$1) \text{ slope: } \frac{42-34}{-6--5} = \frac{8}{-1} = -8$$

$$y - 10 = -8(x + 2)$$

$$y - 10 = -8x - 16$$

$$y = -8x - 6$$

$$4) \text{ slope: } \frac{8-6}{0-1} = \frac{2}{-1} = -2$$

$$y - 0 = -2(x - 4)$$

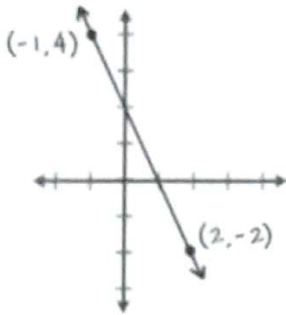
$$y = -2x + 8$$

### Concept 4: Average Rate of Change

What is average rate of change?

Average rate of change is very closely related to slope. In fact, the average rate of change of a line is the slope of the line.

Find the rate of change of the following lines:



x	0	1	2	3	4
y	2	4	6	8	10

$$y = -6x - 5$$

Average Rate of Change:

$$\frac{-2 - 4}{2 - (-1)} = \frac{-6}{3} = -2$$

Average Rate of Change:

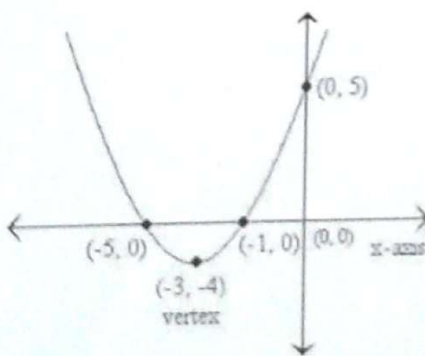
$$\frac{4 - 2}{1 - 0} = \frac{2}{1} = 2$$

Average Rate of Change:

$$-6$$

However, when you are finding the average rate of change of functions that are not linear, they don't have a consistent slope throughout. Because of this, we find the average rate of change across a certain interval. For example, we'll find the average rate of change from  $[0, 2]$ . Note that when writing in set notation,  $[$  means the value is included,  $($  means the value is not included.

To do this, we simply find the slope between the two x-values given in the interval. To find the slope, we need two points so we must find the y-values for each of the x-values either from a table, graph, set of ordered pairs, or an equation. Find the average rate of change of the following three non-linear functions on their given intervals.



x	y
1	1
2	4
3	9
4	16
5	25

$$y = -(x - 1)^2 + 5$$

Interval:  $[0, 1]$

Estimate: 5

Average Rate of Change:

$$\frac{5 - 0}{0 - (-1)} = \frac{5}{1} = 5$$

Interval:  $[2, 3]$

Average Rate of Change:

$$\frac{9 - 4}{3 - 2} = \frac{5}{1} = 5$$

Interval:  $[0, 2]$

Average Rate of Change:  $(0, 4)$

$$\frac{4 - 4}{0 - 2} = \frac{0}{-2} = 0 \quad (2, 4)$$