Concept 1: Simplifying, Multiplying, and Dividing Rational Expressions

Simplify the following fractions.

1. $\dfrac{-16}{24} \div \dfrac{8}{8} = \dfrac{-2}{3}$

E.V. none

2. $\dfrac{x^3}{x} \div \dfrac{x}{x} = \dfrac{x^2}{1} = x^2$

E.V. $x=0$

3. $\dfrac{12x^2}{48x^4} \div \dfrac{12x^2}{12x^2} = \dfrac{1}{4x^2}$

E.V. 0

4. $\dfrac{2x+2}{3x+3} = \dfrac{2(x+1)}{3(x+1)} = \dfrac{2}{3}$

E.V. -1

5. $\dfrac{2x+4}{x^2-2x-8} = \dfrac{2(x+2)}{(x+2)(x-4)}$

$\dfrac{2}{x-4}$ $\begin{array}{l}A-8\\B-2\end{array}$

E.V. $-2, 4$

6. $\dfrac{x^2+6x+8}{x^2+10x+16} = \dfrac{(x+2)(x+4)}{(x+2)(x+8)}$

$\dfrac{x+4}{x+8}$

E.V. $= -8, -2$

What is an excluded value?

---Any x-value that would cause you to divide by 0. (i.e. any x-value that would give you a 0 in the denominator)

How do you find an excluded value?

---Set the denominator equal to 0 and solve.

Find the excluded values for problems 1-6.

Multiply the following fractions. State the excluded values.

1. $\dfrac{2}{5} \times \dfrac{3}{4} = \dfrac{6}{20} = \dfrac{3}{10}$

E.V. none

2. $\dfrac{2x}{3} \times \dfrac{6}{x} = \dfrac{12x}{3x} = 4$

E.V. 0

3. $\dfrac{x+6}{x-3} \times \dfrac{x-3}{x-2} = \dfrac{(x+6)(x-3)}{(x-3)(x-2)}$

$\dfrac{x+6}{x-2}$

E.V. $3, 2$

4. $\dfrac{3x+12}{4x} \times \dfrac{4}{(x+4)(x-2)}$

$\dfrac{3(x+4)}{4x} \times \dfrac{4}{(x+4)(x-2)}$

$\dfrac{12(x+4)}{4x(x+4)(x-2)} = \dfrac{3}{x(x-2)}$

E.V. $x \neq 0, 2, -4$

5. $\dfrac{x^2+4x+4}{x+7x+12} \times \dfrac{5x+15}{2} = \dfrac{(x+2)(x+2)5(x+3)}{(x+3)(x+4)2}$

$\dfrac{5(x+2)^2}{2(x+4)}$

3

How do you divide fractions?

1. $\frac{3}{4} \div \frac{2}{3}$ = $\frac{3}{4} \times \frac{3}{2}$ = $\frac{9}{8}$ 2. $\frac{4}{5} \div \frac{x}{2}$ = $\frac{4}{5} \times \frac{2}{x}$ = $\frac{8}{5x}$ 3. $\frac{3x+6}{x-1} \div \frac{x+2}{x-5}$ $\frac{3(x+2)}{x-1} \times \frac{x-5}{x+2}$

$$\frac{3(x+2)(x-5)}{(x-1)(x+2)} = \frac{3(x-5)}{(x-1)}$$

How does this affect your excluded values? Consider #3 written like this: $\dfrac{\frac{3x+6}{x-1}}{\frac{x+2}{x-5}}$

What are the excluded values for #3?

$x-1=0$ $x+2=0$ $x-5=0$

E.V. $x \neq 1, -2, 5$

Try the following examples, State the excluded values:

4. $\dfrac{x^2-8x+15}{x^2-x-6} \div \dfrac{x^2+x-20}{x^2-6x+8}$ 5. $\dfrac{x^2+2x-15}{x^2-4x-45} \div \dfrac{x^2+x-12}{x^2-5x-36}$

$$\frac{(x-5)(x-3)}{(x-3)(x+2)} \div \frac{(x+5)(x-4)}{(x-2)(x-4)} \qquad \frac{(x-3)(x+5)}{(x-9)(x+5)} \div \frac{(x-3)(x+4)}{(x-9)(x+4)}$$

$$\frac{(x-5)(x-3)}{(x-3)(x+2)} \times \frac{(x-2)(x-4)}{(x+5)(x-4)} \qquad \frac{(x-3)}{(x-9)} \times \frac{(x-9)}{(x-3)}$$

$$\frac{(x-5)(x-3)(x-2)(x-4)}{(x-3)(x+2)(x+5)(x-4)} \qquad \frac{(x-3)(x-9)}{(x-9)(x-3)}$$

$$\boxed{\frac{(x-5)(x-2)}{(x+2)(x+5)}} \qquad \boxed{1}$$

E.V. $3, -2, -5, 4, 2$ E.V. $9, -5, 3, -4$

4

Concept 2: Adding and Subtracting Rational Expressions

How do you add and subtract fractions?

1. $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$

2. $\frac{3}{8} - \frac{1}{4} = \frac{3}{8} - \frac{2}{8} = \frac{1}{8}$

3. $\frac{2}{5} + \frac{3}{7} = \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$

Just like you have to have a common denominator when adding and subtracting fractions, you have to have a common denominator when adding rational expressions.

Try adding the following expressions that already have a common denominator.

4. $\frac{3x+1}{2x^2} + \frac{2x-5}{2x^2} = \frac{5x-4}{2x^2}$

E.V. 0

5. $\frac{3x+2}{x+1} - \frac{2x+1}{x+1} = \frac{x+1}{x+1} = 1$

E.V. -1

How do you find a common denominator?

PUT ALL DENOMINATORS IN THEIR MOST FACTORED FORM!!!

Find a common denominator for the following, then add or subtract and simplify.

6. $\frac{x-3}{3x} - \frac{x-5}{x} \cdot \frac{3}{3} = \frac{x-3}{3x} - \frac{3x-15}{3x}$

C.D.- $3x$

$\frac{-2x+12}{3x}$ or $\frac{-2(x-6)}{3x}$

7. $\frac{x+2}{x-1} + \frac{x}{4x-4}$ $\frac{4x+8}{4x-4} + \frac{x}{4x-4} = \frac{5x+8}{4x-4}$

C.D.- $4(x-1)$

$\frac{5x+8}{4x-4}$ OR $\frac{5x+8}{4(x-1)}$

8. $\frac{x}{3x+6} - \frac{x-5}{2x+4}$ $\frac{2}{2} \cdot \frac{x}{3(x+2)} - \frac{x-5}{2(x+2)} \cdot \frac{3}{3}$

C.D.- $6(x+2)$

$\frac{2x}{6(x+2)} - \frac{3x-15}{6(x+2)} = \frac{-x-15}{6(x+2)}$

9. $\frac{(x+2)(x-4)}{x-1} + \frac{x}{3(x-4)}$

C.D.- $3(x-4)(x-1)$

10. $\frac{2x-15}{x^2-4x-12} + \frac{x^2+x-12}{x^2-36}$

C.D.- $(x-6)(x+6)(x+2)$

$\frac{(x+2)2x-15}{(x+2)(x-6)(x+2)} + \frac{x^2+x-12}{(x-6)(x+6)} \cdot \frac{(x+2)}{(x+2)}$

$\frac{2x^2-15x+12x-30}{(x+6)(x-6)(x+2)} + \frac{x^3+2x^2+x^2+2x-12x-24}{(x+6)(x-6)(x+2)}$

11. $\frac{x^2-8x+15}{x^2+x-6} - \frac{x^2+x-20}{x^2-6x+8}$ $\frac{(x-4)x^2-8x+15}{(x-4)(x-2)(x+3)} - \frac{x^2+x-20(x+3)}{(x-2)(x-4)(x+3)}$

C.D.- $(x-2)(x+3)(x-4)$

$\frac{x^3-8x^2+15x-4x^2+32x-60}{(x-2)(x+3)(x-4)} - \frac{x^3+x^2-20x+3x^2+3x-60}{(x-2)(x+3)(x-4)}$

$\frac{x^3+5x^2-13x-54}{(x+6)(x-6)(x+2)}$ $-8x^2+30x^2$

5

Concept 3: Solving Rational Equations

REMINDER: An extraneous solution is a solution you get as a result of solving the equation but that doesn't "work" or make the equation true if you plug it back in. Extraneous solutions are NOT real solutions and should not be included in your final answer. Just like when solving radical equations, you should ALWAYS check for extraneous solutions when solving rational equations.

Solving rational equations simply combines what you already know about solving other types of equations and adding and subtracting rational expressions. Let's try it!

1) $\dfrac{1}{m} - \dfrac{5}{2} = \dfrac{1}{2m}$ C.D. $2m$

$$\dfrac{m}{2m} - \dfrac{10}{2m} = \dfrac{1}{2m}$$

$$2m \cdot \dfrac{m-10}{8m} = \dfrac{1}{2m} \cdot 2m$$

$$m - 10 = 1$$

$$\boxed{m = 11}$$

2) $\dfrac{1}{2x+1} = \dfrac{1}{x2} - \dfrac{1}{2x}$ C.D. $2x$

$$\dfrac{2x}{2x} = \dfrac{2}{2x} - \dfrac{x}{2x}$$

$$2x = 2 - x$$
$$+x \qquad +x$$
$$\dfrac{3x}{3} = \dfrac{2}{3}$$

$$\boxed{x = \dfrac{2}{3}}$$

3) $\dfrac{1}{2m^2} + \dfrac{m-2}{4m^2} = \dfrac{m+2}{4m^2}$ C.D. $4m^2$

$$\dfrac{2}{4m^2} + \dfrac{4m-8}{4m^2} = \dfrac{m+2}{4m^2}$$

$$2 + 4m - 8 = m + 2$$
$$-6 + 4m = m + 2$$
$$\quad +6 \qquad -m \quad -m \quad +6$$
$$\dfrac{3m}{3} = \dfrac{8}{3}$$

$$\boxed{m = \dfrac{8}{3}}$$

4) $\dfrac{5}{4b} + \dfrac{3}{b^2} = \dfrac{1}{4b^2}$

$$\dfrac{5b}{4b^2} + \dfrac{12}{4b^2} = \dfrac{1}{4b^2}$$

$$5b + 12 = 1$$
$$\quad -12 \quad -12$$

$$\boxed{b = \dfrac{-11}{5}}$$

5) $\dfrac{5}{a^2 + 2a} = \dfrac{a-5}{3a^2 + 6a} - \dfrac{1}{3a^2 + 6a}$

$$3a(a+2) \quad 3a(a+2) \quad 3a(a+2)$$

$$\dfrac{15}{3a(a+2)} = \dfrac{a-5}{3a(a+2)} - \dfrac{1}{3a(a+2)}$$

$$15 = a - 5 - 1$$
$$15 = a - 6$$
$$+6 \qquad +6$$

$$\boxed{a = 21}$$

6) $\dfrac{6}{x^2 - 4x + 3} = \dfrac{1}{x^2 - 4x + 3} + \dfrac{1}{x-3}$ (x-1)

$$(x-3)(x-1) \quad (x-3)(x-1)$$

$$\dfrac{6}{(x-3)(x-1)} = \dfrac{1}{(x-3)(x-1)} + \dfrac{x-1}{(x-3)(x-1)}$$

$$6 = x$$

$$\boxed{x = 6}$$

Concept 4: Solving Rational Inequalities

What is an interval?

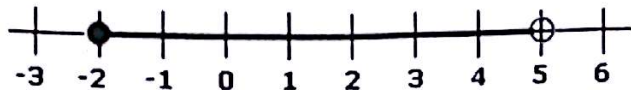All the numbers that lie between two numbers.

Included: ≤ ≥ [ ] •

Not Included: < > ( ) ○

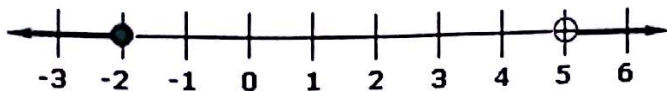Write the intervals for the following number lines using both inequality notation and interval notation.
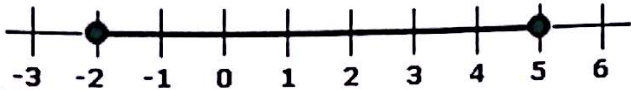

Graph 1 $(-\infty, -2)$ $(5, \infty)$

Graph 2 $[-2, 5)$

Graph 3 $(-\infty, -2]$ $(5, \infty)$

Graph 4 $[-2, 5]$

What are critical values and how do you find them?

-Critical values are any x-values that would make the numerator OR the denominator equal to 0.

Thus, to find them you factor the numerator and denominator and set every factor equal to 0 and solve.

Find the critical values of the following rational inequalities:

5. $\dfrac{x^2-9}{5-x} \leq 0$

$x^2 - 9 = 0$

$\sqrt{x^2} = \sqrt{9}$

$-3, 3, 5$ $x = \pm 3$

$5 - x = 0$

$x = 5$

6. $\dfrac{1}{(x+5)(4-x)} \geq 0$ $-5, 4$

7. $\dfrac{x+4}{x-4} < 0$ $-4, 4$
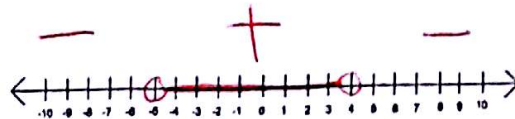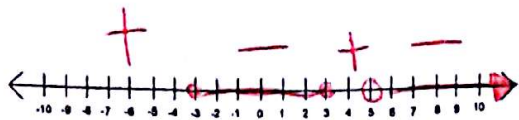
8. $\dfrac{x+2}{2x+6} \leq 0$ $-2, -3$

7
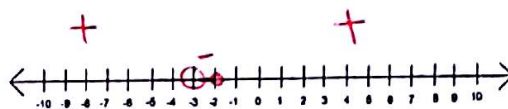
Plot the critical values of each problem on a number line.

5. $\dfrac{x^2-9}{5-x} \le 0$ $\quad$ -3, 3, 5

6. $\dfrac{1}{(x+5)(4-x)} \ge 0$ $\quad$ -5, 4

Interval: $[-3, 3] \ (5, \infty)$

Interval: $(-5, 4)$

7. $\dfrac{x+4}{x-4} < 0$ $\quad$ -4, 4

8. $\dfrac{x+2}{2x+6} \le 0$ $\quad$ -2, -3

Interval: $(-4, 4)$

Interval: $(-3, -2]$

In order to find the intervals, test one number between each set of critical values and decide if it is positive or negative. Then you can see in which intervals your expression holds true.

In the space below, write a positive growth mindset message about math!!! ☺

Doing HARD things makes your BRAIN grow the MOST!

8