Unit 6: Quadratics

(Vertex, Intercepts, and Zeroes)

Guided Notes

KEY

Name

Period

If found, please return to Mrs. Brandley's room, M-8.

Concept 1: WARM-UP

Simplify the following:

1. $(x+2)^2$

$(x+2)(x+2)$

x^2+4x+4

2. $(x-5)^2$

$(x-5)(x-5)$

$x^2-10x+25$

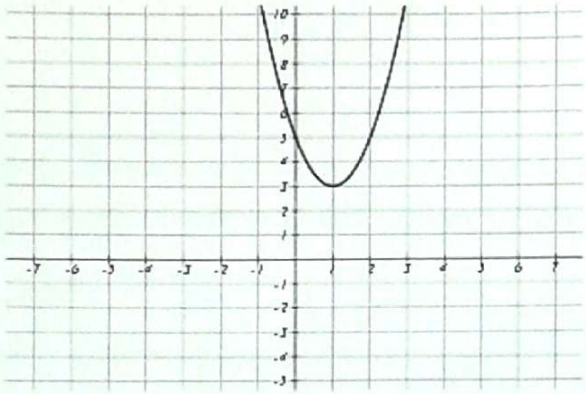
3. $(x+4)^2$

$(x+4)(x+4)$

$x^2+8x+16$

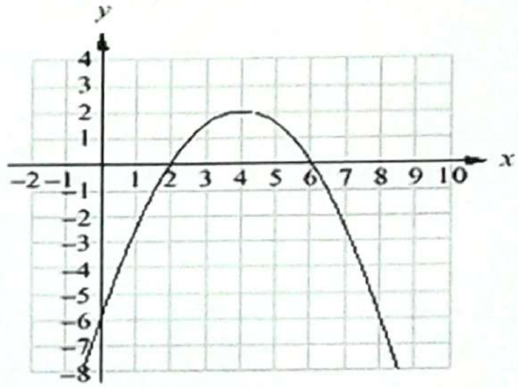
State the axis of symmetry and the vertex of the following graphs of quadratics:

5.



Axis of Symmetry: $x=1$ Vertex: $(1,3)$

6.



Axis of Symmetry: $x=4$ Vertex: $(4,2)$

State the transformations of the following functions:

7. $y=-(x-3)^2+7$

Flipped over x-axis
right 3
up 7

8. $y=(x+2)^2+5$

left 2
up 5

9. $y=(-x-6)^2-4$

Flipped over y-axis
right 6
down 4

Concept 1: Changing Forms and Vertex

Standard Form: $y=ax^2+bx=c$

Vertex Form: $y=a(x-h)+k$

Identify which form the following quadratics are in:

10. $y=4(x+3)^2+1$ vertex

11. $y=2x^2-3x+7$ Standard

12. $y=4x^2-7x+9$ Standard

13. $y=3(x-2)^2-8$ vertex

In vertex form, (h,k) is the vertex of the quadratic. List the vertex of numbers 10 and 13 above.

10. $(-3,1)$

13. $(2,-8)$

The formula to find the axis of symmetry algebraically is $x = \frac{-b}{2a}$. Find the axis of symmetry for 11 and 12.

11. $y = 2x^2 - 3x + 7$

12. $y = 4x^2 - 7x + 9$

X= $\frac{--3}{2(2)} = \frac{3}{4}$

X= $\frac{--7}{2(4)} = \frac{7}{8}$

Notice that the axis of symmetry, is also the x-value of the vertex. So in order to find the vertex, plug the axis of symmetry value in for x and solve for y. Find the vertex for numbers 5 and 6.

11. $2(\frac{3}{4})^2 - 3(\frac{3}{4}) + 7 = 5.875$

$(3/4, 5.875)$

12. $4(\frac{7}{8})^2 - 7(\frac{7}{8}) + 9 = 5.9375$

$(7/8, 5.9375)$

Now that you know the vertex for numbers 11 and 12, and the a value can be found from the quadratic in standard form, write numbers 11 and 12 in vertex form. $y = a(x - h) + k$

11.

$Y = 2(x - 3/4) + 5.875$

12.

$Y = 4(x - 7/8) + 5.9375$

Write numbers 10 and 13 in standard form by simplifying the function.

10. $y = 4(x + 3)^2 + 1$

$(x+3)(x+3)$

$4(x^2 + 6x + 9) + 1$

$4x^2 + 24x + 36 + 1$

$4x^2 + 24x + 37$

13. $y = 3(x - 2)^2 - 8$

$(x-2)(x-2)$

$3(x^2 - 4x + 4) - 8$

$3x^2 - 12x + 12 - 8$

$3x^2 - 12x + 4$

Put the following two functions in vertex form.

14. $y = x^2 - 6x + 2$

$X = \frac{--6}{2(1)} = \frac{6}{2} = 3$

$3^2 - 6(3) + 2$

$9 - 18 + 2$

-7 $(3, -7)$

$Y = (x-3)^2 - 7$

15. $y = -2x^2 + 12x + 5$

$X = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$

$-2(3^2) + 12(3) + 5$ vertex: $(3, 23)$

$-2(9) + 36 + 5$

$-18 + 36 + 5 = 23$

$Y = -2(X-3) + 23$

Concept 2: WARM-UP

Find the axis of symmetry and the vertex of the following functions:

1. $y = 2(x-3)^2 + 1$

$2(x^2-6x+9)+1$

$2x^2-12x+18+1$

$2x^2-12x+19$

$x = \frac{12}{4} = 3$

$\boxed{\begin{matrix} x=3 \\ (3,1) \end{matrix}}$

2. $y = 3x^2 - 6x$

$x = \frac{6}{2(3)} = \frac{6}{6} = 1$

$\boxed{\begin{matrix} x=1 \\ (1,-3) \end{matrix}}$

3. $y = -2x^2 + 2x + 4$

$x = \frac{-2}{2(-2)} = \frac{-2}{-4} = \frac{1}{2}$

$\boxed{\begin{matrix} x=\frac{1}{2} \\ (\frac{1}{2}, 4\frac{1}{2}) \end{matrix}}$

Concept 2: Intercepts and Zeroes

Find the axis of symmetry, vertex, y-intercept, and two additional points of each function and use them to graph the function. Then list the solutions of each function.

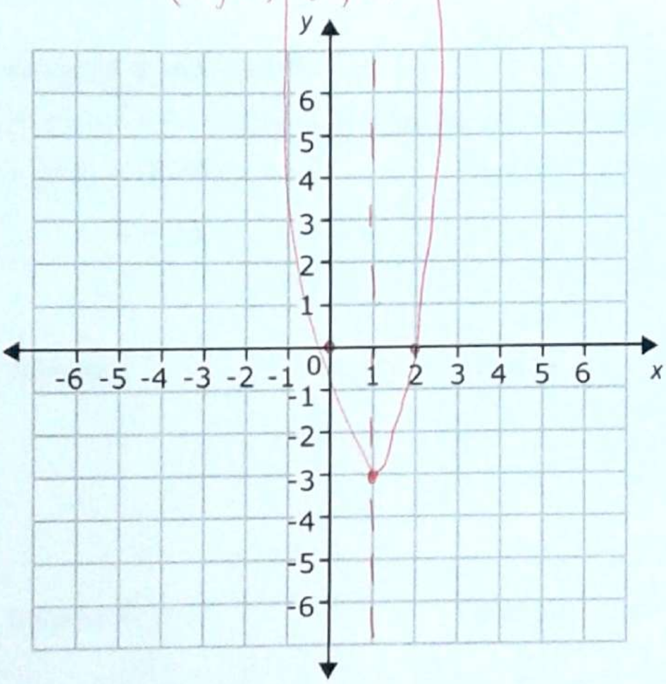
$y = 3x^2 - 6x$

Axis of symmetry: $x = 1$

Vertex: $(1, -3)$

Y-Intercept: $(0, 0)$

Two Points: $(2, 0)$ $(4, 9)$



Solutions: $x = 0, 2$

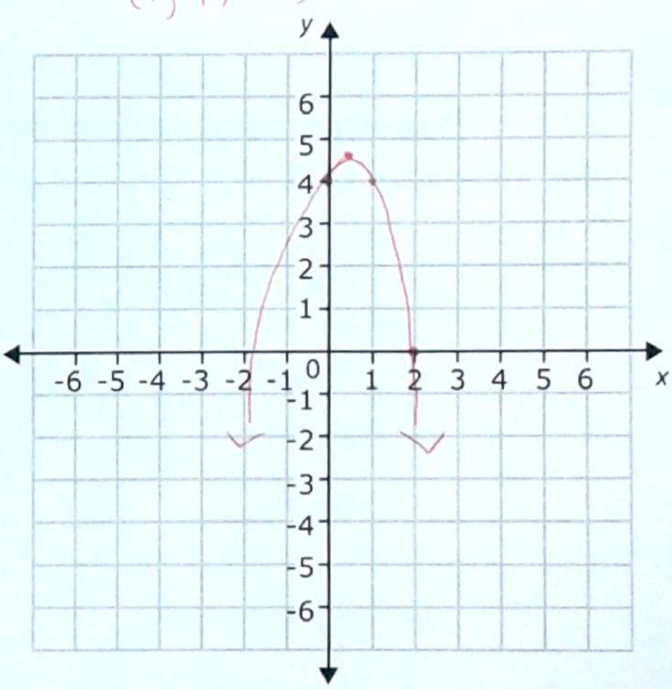
$y = -2x^2 + 2x + 4$

Axis of symmetry: $x = \frac{1}{2}$

Vertex: $(\frac{1}{2}, 4\frac{1}{2})$

Y-Intercept: $(0, 4)$

Two Points: $(1, 4)$ $(2, 0)$



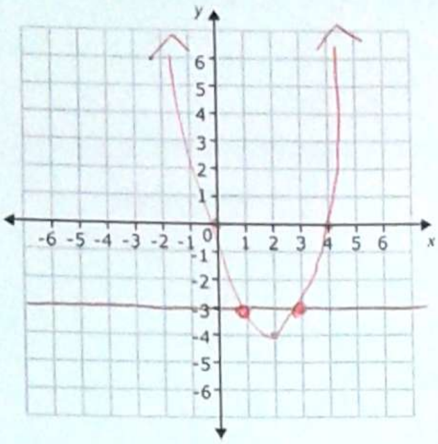
Solutions: $x = -2, 2$

How can you find the real solutions of a quadratic by looking at its' graph? Why is that the case?

Finding the x-intercept x-values because that is what x equals when $y=0$.

Using a graphing calculator, graph each side of the equal sign as a separate function and sketch the graphs below. The intercepts of the two functions give the solutions to the function. List the solutions below.

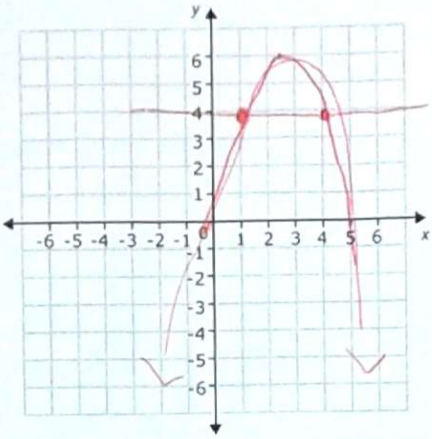
1. $x^2 - 4x = -3$



Solutions:

$x = 1, 3$

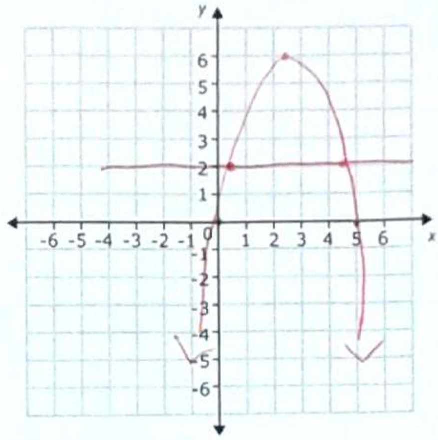
2. $-x^2 + 5x = 4$



Solutions:

$x = 1, 4$

3. $-x^2 + 5x = 2$



Solutions:

$x = \frac{1}{2}, 4\frac{1}{2}$

Concept 3: WARM-UP

For each of the following, identify the zeroes, find the standard form equation and the vertex.

1. $f(x) = (x-2)(x+3)$

Zeroes: $x - 2 = 0 \quad x + 3 = 0$

$x = 2 \quad x = -3$

$\boxed{x = 2, -3}$

Standard Form:

$x^2 + x - 6$

Vertex:

$x = \frac{-1}{2(1)} = -\frac{1}{2}$

$(-\frac{1}{2})^2 + -\frac{1}{2} - 6 = -6\frac{1}{4}$

$(-\frac{1}{2}, -6\frac{1}{4})$

2. $f(x) = 2(x-4)(x+1)$

Zeroes: $x - 4 = 0 \quad x + 1 = 0$

$x = 4 \quad x = -1$

$\boxed{x = 4, -1}$

Standard Form:

$2x^2 - 6x - 8$

Vertex:

$x = \frac{6}{2(2)} = \frac{6}{4} = \frac{3}{2}$

$2(\frac{3}{2})^2 - 6(\frac{3}{2}) - 8 = -12\frac{1}{2}$

$(3/2, -12\frac{1}{2})$

3. $f(x) = -3(x+5)(x-3)$

Zeroes: $x + 5 = 0 \quad x - 3 = 0$

$x = -5 \quad x = 3$

$\boxed{x = -5, 3}$

Standard Form:

$-3x^2 - 6x + 45$

Vertex:

$x = \frac{6}{2(-3)} = \frac{6}{-6} = -1$

$-3(-1)^2 - 6(-1) + 45 = 48$

$(-1, 48)$

Concept 3: Intercepts and Linear Factors

What are the different ways we have seen for finding solutions to quadratic equations?

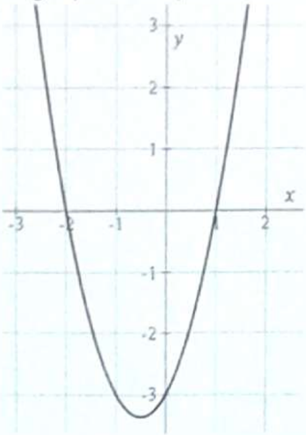
-Solving for zeroes in factored form

Try this one! $(x-3)(x+7)$

X=3, -7

-Looking for the x-intercepts in the graph of a quadratic

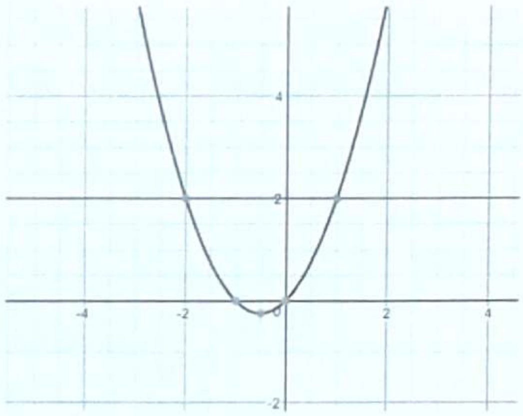
Try this one!



X=-2, 1

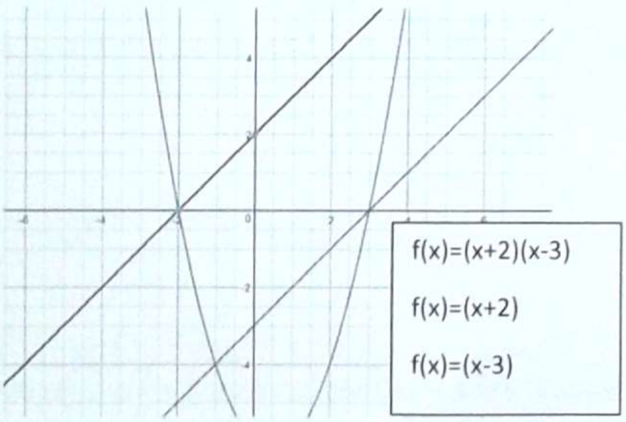
-Finding the intersection of the function on either side of the equal sign

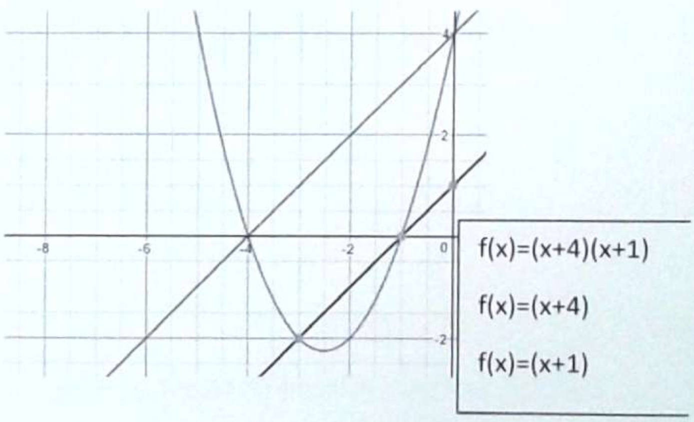
Try this one! $x^2+x=2$



X=-2, 1

-(This one is new) Graph each linear factor separately and see where their x-intercepts fall in comparison to the x-intercepts of the quadratic. Look for patterns in these two.





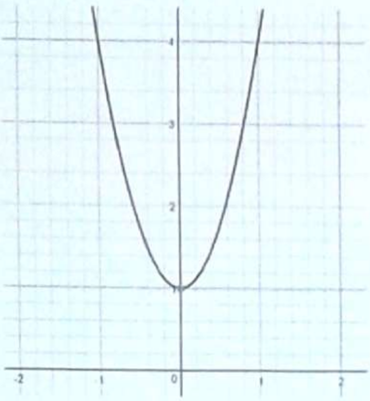
X=-2, 3

X=-4, -1

Concept 4: WARM-UP

State how many total, complex, and real solutions the following functions have:

1. $y = 3x^2 + 1$ —

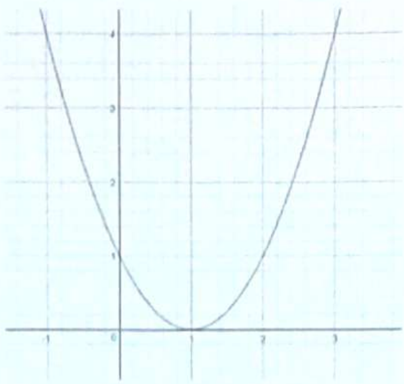


Total: 2

Real: 0

Complex: 2

2. $y = (x-1)^2$ 0

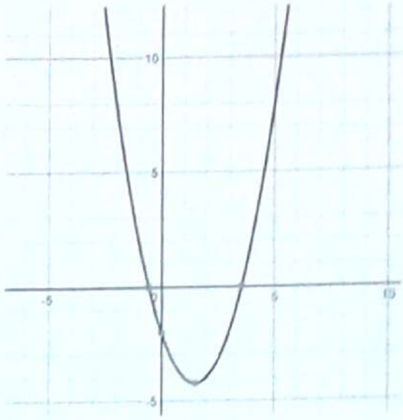


Total: 2

Real: 2 (repeat)

Complex: 0

3. $y = x^2 - 3x - 2$ +



Total: 2

Real: 2

Complex: 0

Concept 4: Number and Types of Solutions

We learned how to find the number and types of solutions of functions when given both their equation and their graph. Today we will learn how to find the type of solutions from just the equation. The key to doing so is the discriminant.

Discriminant: $b^2 - 4ac$

Positive Discriminant: 2 real solutions

Negative Discriminant: 2 complex solutions

Zero Discriminant: A repeated solution

Find the discriminant of the following functions and state whether they have 2 real solutions, 2 complex solutions, or a repeated solution.

4. $y = 3x^2 - 6x + 1$

$(-6)^2 - 4(3)(1)$

$36 - 12$

24

2 real solutions

5. $y = x^2 - 4x + 4$

$(-4)^2 - 2(1)(4)$

$16 - 8$

8

2 real solutions

6. $y = -x^2 - 3x - 7$

$(-3)^2 - 4(-1)(-7)$

$9 - 28$

-19

2 complex solutions

Go back to the 3 warm-up questions and state whether the discriminant would be positive, negative, or zero.